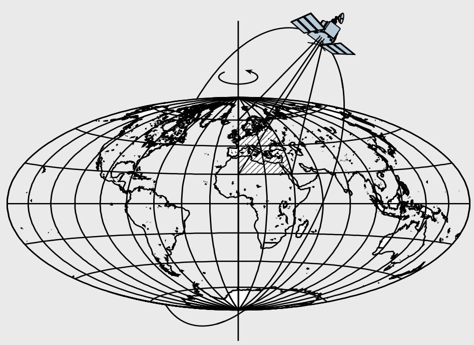
# INS/GPS Vector Gravimetry Along Roads in Western Montana

by

Christopher Jekeli and Xiaopeng Li



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Geodetic Science and Surveying
Department of Geological Sciences
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# INS/GPS Vector Gravimetry Along Roads in Western Montana

## A First Comprehensive Report

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Division of Geodetic Science
Department of Geological Sciences
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## Acknowledgments

The survey could not have been done without the generosity of OSU's Center for Mapping in allowing us to use the GPSVan. Charles Toth (of the Center for Mapping) offered his time to drive the van to Montana and back (helped by two students not directly associated with this project, Eva Paska and Nora Csanyi), as well as to conduct the actual surveys. In addition, he was responsible for integrating the system instruments and requisite computers and data loggers, and he proved indispensable in ensuring that they were properly functioning (at least recording data) each day. Our gratitude also goes to NGA personnel, Robert Anderson, Steve Kenyon, Terry Timblin, and Glen Noll for the time they gave to take care of the GPS base stations during the surveys. We also acknowledge the help from student, Yudan Yi (also at the Center for Mapping), in creating usable data sets for the LN100, and to student, Jong Ki Lee, for performing the GEOCOL estimations of the DOV. Finally, we thank Prof. Dorota Brzezinska (Department of Civil and Environmental Engineering and Geodetic Science) for letting us use the Applanix™ GPS software, as well as allowing her students (Paska, Csanyi, Yi) to help with the project.

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# Executive Summary

This document reports on a comprehensive look at the data collected by the GPSVan (OSU, Center for Mapping) in western Montana in April and June of 2005. The data consist of inertial measurement unit (IMU) data, extracted from high-accuracy inertial navigation systems, and differential GPS data that are combined to estimate the (3-D) gravity vector along the roadways traveled by the vehicle. The key to the evaluation of these tests and to their deemed success was the repeated runs of the traverses, rather than the existing control data in the region. The fairly dense network of gravity data provided only some overall corroboration of the accuracy in the vertical components of the estimates. The deflection of the vertical (DOV) data and other independent sources of computed DOV's provided barely some long-wavelength confirmation of our estimates, while the repeatability of the traverses verified fine detail in the recovered horizontal components. However, this precision was not consistent and large errors remain in the horizontal components. The single largest detriment to our estimates was the inaccuracy in the kinematic GPS positioning solution. Due to road overpasses and other obstructions, the GPS solution was often degraded significantly due to the inability to solve for the cycle ambiguity. This had a direct and demonstrable effect on the gravity estimation. When all systems were working at peak performance, we showed better than mgal repeatability in the down component of the gravity disturbance and standard deviation of 2-3 mgal with respect to the interpolated control data. No attempt was made in this first analysis to solve for biases and linear trends, nor to take advantage of the multiple traverses to arrive at final along-track gravity disturbance estimates. Three essential conclusions were obtained from our analysis: 1) GPS solutions must be improved, e.g., using INS to help recover the cycle ambiguity after a GPS outage; 2) more direct, along-track control data are necessary, particularly in the horizontal components, to obtain a meaningful assessment of the vector gravimetry capability of the system; and 3) an operational system would clearly benefit from redundancy in instrumentation in order to imitate and take advantage of multiple traverses along each surveyed road.

The first chapter summarizes the instrument setup, the survey routes, the data collected, and the control data available. The second chapter briefly reviews the techniques used to obtain the gravity vector estimates, relying heavily on previous publications and reports. Results of applying these techniques to the data are shown in Chapter 3; followed by the concluding chapter with comments and analyses, and an outlook toward further data processing.

# 1. Instrumentation and Data Collection

## 1.1 Instrument Setup and Survey Routes

The survey vehicle is a GMC Suburban modified for GIS-type surveys (2 GPS antennas mounted on the roof, camera mounts available, and the interior is outfitted with a secure instrument platform and battery-driven power supplies). This vehicle, known as the GPSVan, belongs to OSU's Center for Mapping and was kindly provided to conduct the gravity survey tests. Figure 1 shows the GPSVan and the interior suite of IMU and GPS instrument, looking aft. Table 1 lists in more detail the types of instruments used during the mobile gravity surveys in Montana. It should be noted that only the high-accuracy IMU's, contained in the inertial navigation systems H764G and LN100, were analyzed for the gravity estimation in this report. Data from the HG1700 and LN200, may also yield some useful results, but these have not yet been analyzed. Data from the Crossbow 400CC are not of sufficient quality to attempt gravity estimation.

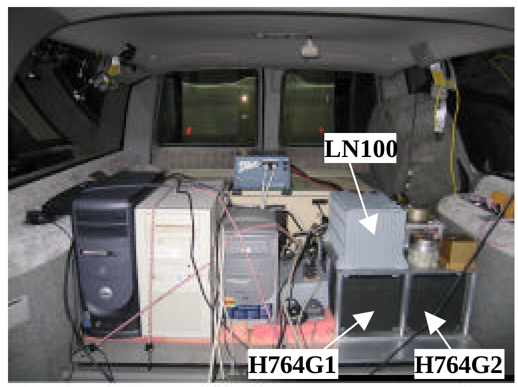



Figure 1: GPSVan (left) and interior suite of computers and IMU's (only the indicated IMU's were used in the analyses of this report).

Table 1: Details of instrumentation used on Montana Surveys

| **Type** | **Model** | **Name** | **Manufacturer** | **utilization** |
|---|---|---|---|---|
| INS | H764G | H764G1 | Honeywell | IMU data and nav solution |
| INS | H764G | H764G2 | Honeywell | IMU data and nav solution |
| INS | LN100 | LN100 | Litton | IMU data and nav solution |
| IMU | HG1700 | -- | Honeywell | not used in data analysis |
| IMU | LN200 | -- | Litton | not used in data analysis |
| IMU | 400CC | -- | Cross Bow | not used in data analysis |
| GPS receiver | OEM4 | NovAtel1 | Novatel | rover |
| GPS receiver | OEM4 | NovAtel2 | Novatel | rover spare |
| GPS receiver | OEM4 | NovAtel3 | Novatel | rover |
| GPS receiver | 5700 | Trimble1 | Trimble | rover spare |
| GPS receiver | 5700 | Trimble2 | Trimble | rover |
| GPS receiver | 5700 | Trimble3 | Trimble | rover spare |
| GPS receiver | 5700 | Trimble4 | Trimble | rover spare |
| GPS receiver | 4000ssi | Trimble5 | Trimble | rover time synchronization |
| GPS receiver | Legacy-E | Topcon1 | Topcon | rover |
| GPS receiver | Legacy-E | Topcon2 | Topcon | base station |
| GPS receiver | Ashtech z12 | Ashtech1 | Ashtech | base station (NGA) |
| GPS receiver | Ashtech z12 | Ashtech2 | Ashtech | base station (NGA) |
| GPS receiver | Ashtech z12 | Ashtech3 | Ashtech | base station (NGA) |
| GPS receiver | Ashtech z12 | Ashtech4 | Ashtech | base station (NGA) |

Figure 2 shows the routes traveled by the GPSVan in western Montana. A preliminary trial run was made on 28 April 2005 along interstate route I-90 between Butte and Missoula, MT. More extensive surveys were conducted on 13-15 June 2005, extending over some mountain passes and through major valleys, and passing by previously surveyed deflection of the vertical (DOV) points. In all cases, the vehicle essentially remained on well-paved roads (with a few small excursions to the actual DOV points). The planned vehicle sorties were designed to test continuous, as well as stop-and-go travel. Passing by or occupying DOV points, besides providing some local control, would also enable testing of waypoint densification along connecting routes. Vehicle speed was at or below posted limits. All roads were surveyed at least twice, with the I-90 segment traversed a total of four times, with one additional sub-segment. In addition to the permanently established NGS CORS station at Missoula, a number of GPS base stations were set up temporarily along the roads to support (post-mission) differential GPS positioning.

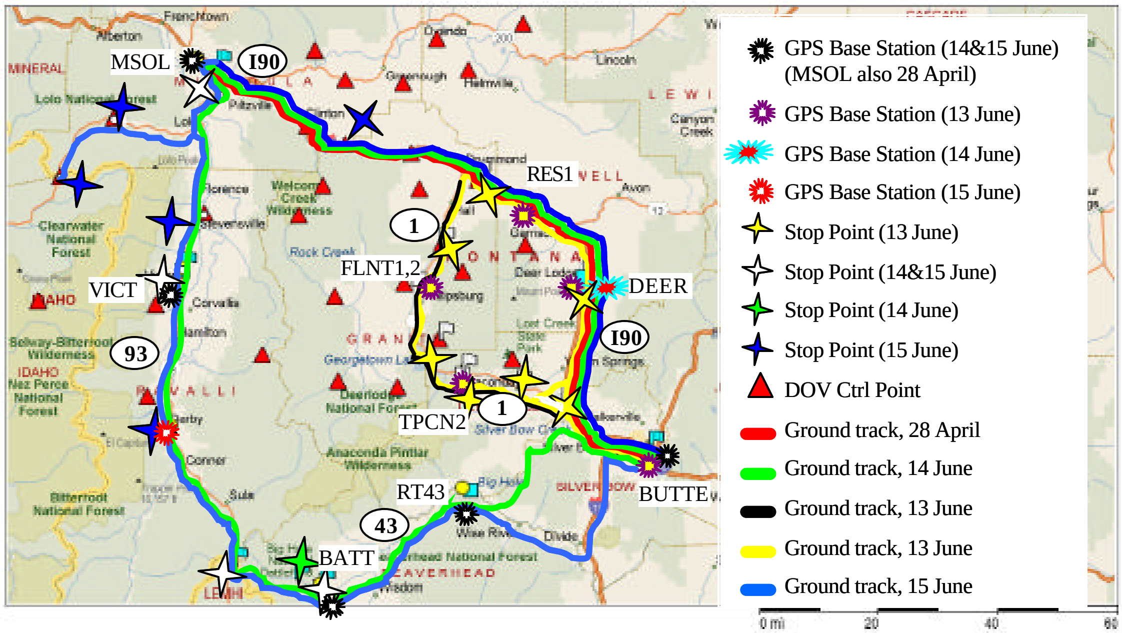



Figure 2: Routes traveled by GPSVan in Montana.

Figure 3 shows points were gravity and DOV data were obtained previously and independently by the National Geospatial-Intelligence Agency (and other agencies) with gravimeters and astrometric instruments (astrolabe and theodolite). Few if any of these points are directly on the roads traversed by the GPSVan; however, it is expected that those points within a few tens of meters of the road can be used as calibration and comparison points without concern for model/interpolation error (only observation error). In addition, the National Geodetic Survey (NGS) produced a 1'×1' DOV grid derived from a national geoid model (GEOID99; Smith and Roman, 2001). This DEFLEC99 model was computed using a two-step procedure. Slopes of GEOID99 (i.e., deflections of the vertical at the geoid) were determined using bicubic spline fits to the geoid; and, subsequently, these were corrected for the curvature of the plumb line based on simple Bouguer gravity anomalies to yield DOV's at the Earth's surface. Table 2 summarizes these control data.

### 1.2 Data Collection on 28 April 2005

Table 3 synopsizes the GPS data collected on 28 April 2005 along the test route I-90 in Montana. There are two tracks in this data set; the first begins in Butte and ends near Missoula to the west; the other follows in reverse (but, of course, on the other side of the divided highway), starting in Missoula and ending in Butte. We used the MSOL Continuously Operating Reference Station (CORS), being the closest GPS base station for this test, to perform differential GPS (DGPS) processing.

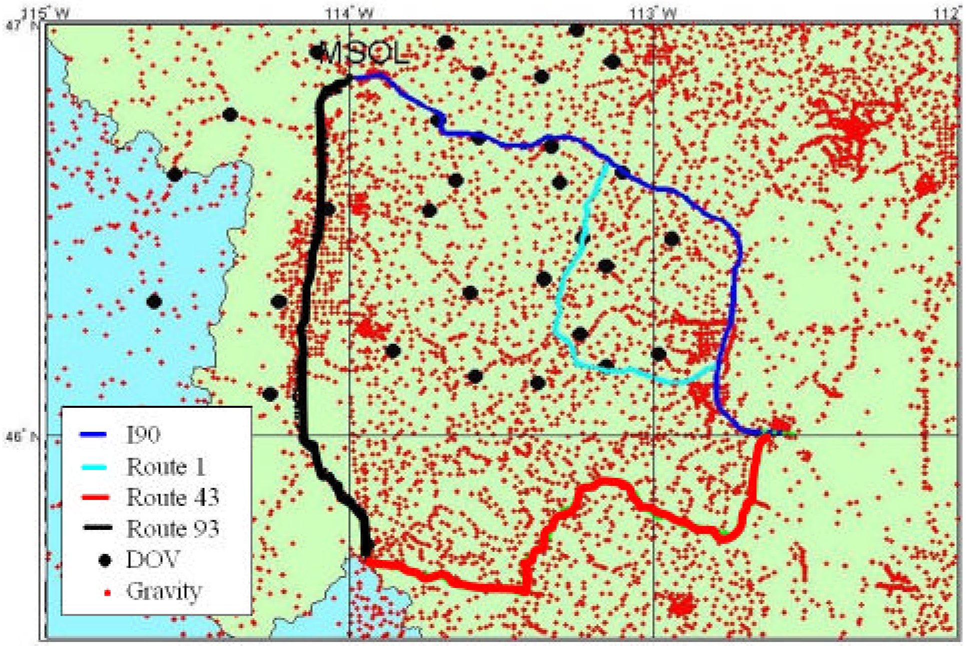



Figure 3: Absolute gravity and DOV/absolute gravity control points surveyed by NGA (black) and other NGA gravity data holdings (red).

Table 2: Summary of control data

| Data Type | number/resolution | accuracy (est) |
|---|---|---|
| Abs. gravity | 31 | 0.01 mgal (est) |
| DOV (astro-geodetic) | 39 | 0.1 arcsec (est) |
| gravity anomaly | 6496 | 1 mgal (est) |
| DOV (DEFLEC99) | 1'×1' | unknown |

Table 3: Rover GPS receiver summary for 28 April 2005. Data rate = 1 Hz. All times are in GPS time (epoch and second of GPS week).

| Receiver | Segment | Start Time | End Time | Data File |
|---|---|---|---|---|
| NovAtel1 | I90 (Butte-Missoula-Butte) | 2005 04 28<br>17 02 55<br>(406975) | 2005 04 28<br>22 14 07<br>(425647) | 00071180.05o |
| NovAtel2 | I90 (Butte-Missoula-Butte) | 2005 04 28<br>17 03 14<br>(406994) | 2005 04 28<br>22 14 20<br>(425660) | 00061180.05o |
| Trimble1 | I90 (Butte-Missoula-Butte) | 2005 4 28<br>17 22 45<br>(408165) | 2005 4 28<br>22 14 45<br>(425685) | 39721180.05o |

Table 4 summarizes the high-end IMU data collected during these two tracks. Unfortunately, H764G2 began to record data only after the vehicle had left Butte, while H764G1 started only after leaving Missoula. This means that these data could not be used directly since they were not preceded by a period of stationary initial alignment (doing a transfer alignment from the other IMU is a possibility that was not explored, since we have sufficient other data for the present analysis). As a result, only four GPS/INS combinations were used to estimate the gravity vector: NovAtel2-H764G1, NovAtel2-LN100 for the westward run and NovAtel2-H764G2, NovAtel2-LN100 for the eastward run.

Table 4: Summary of INS data for 28 April 2005 survey. Data rate = 256 Hz. All times are in GPS time (epoch and second of GPS week).

| **INS** | **Segments** | **Start Time** | **End Time** | **Data File** |
|---|---|---|---|---|
| H764G1 | I90 (Butte-Missoula) | 2005 04 28<br>17 28 13<br>(408493) | 2005 04 28<br>19 37 11<br>(416231) | GM0428_1.SAV |
| | I90 (Missoula-Butte) | 2005 04 28<br>20 04 53<br>(417893) | 2005 04 28<br>22 01 58<br>(424918) | GM0428_1.SAV |
| H764G2 | I90 (Butte-Missoula) | 2005 04 28<br>17 31 35<br>(408695) | 2005 04 28<br>19 38 57<br>(416337) | GM0428_2.SAV |
| | I90 (Missoula-Butte) | 2005 04 28<br>20 01 23<br>(417683) | 2005 04 28<br>22 02 41<br>(424961) | GM0428_2.SAV |
| LN100 | I90 (Butte-Missoula) | 2005 04 28<br>17 11 27<br>(407487) | 2005 04 28<br>19 36 17<br>(416177) | LN1000428_1.BIN |
| | I90 (Missoula-Butte) | 2005 04 28<br>20 11 34<br>(418294) | 2005 04 28<br>22 03 57<br>(425037) | LN1000428_2.BIN |

### 1.3 Data Collection on 13 June 2005

Similar summaries of GPS (base stations and rover) and INS data collections for the 13 June 2005 survey are listed in Tables 5, 6, and 7, respectively, with a graphical comparison shown in Figure 4 of the time spans for most of the instruments. The vehicle was driven essentially non-stop from Butte to Drummond along I-90, with a turn southward on Route 1, and ending in Anaconda. A subsequent traverse of this loop was conducted with occasional stops along the way to re-initialize the INS. INS H764G1 failed to record data during most of this day's traverses; and the data from H764G2 over the I90 segment on the first traverse were inadvertently lost.

With Trimble Geomatics Office (TGO), we used data from CORS base stations MSOL and IDNP (in Idaho) to compute the coordinates of the temporary base stations set up by our survey crew (NGA personnel, see Acknowledgments). The 8 GPS base stations and 4 roving receivers yield a total of 32 different DGPS solutions. However, note that the observation time spans of the base stations and the roving receivers are not all the same; and, we considered only a subset of combinations for the two runs of the Route 1 (SR1) segment (see Section 3.1).

Table 5: Summary of GPS Base Stations set up on 13 June 2005. All times are in GPS time.

| **Station Name** | **Lat/Lon/Ht (WGS84)** | **Begin Time** | **End Time** | **File Name** |
|---|---|---|---|---|
| BUTTE | 45°57'59.96768"N<br>112°30'48.30209"W<br>1663.225m | 2005 06 13<br>12 58 15 | 2005 06 13<br>19 52 27 | BUTT1641.05o |
| FLNT1 | 46°23'54.01455"N<br>113°18'27.06106"W<br>1518.158 m | 2005 06 13<br>15 23 04 | 2005 06 13<br>16 4 59 | FLNT1641.05o |
| FLNT2 | 46°23'54.00591"N<br>113°18'27.06384"W<br>1518.018 m | 2005 06 13<br>17 8 01 | 2005 06 13<br>18 07 59 | FLNT1642.05o |
| TPCN2 | 46°10'07.25022"N<br>113°09'30.92281"W<br>1835.858 m | 2005 06 13<br>16 15 14 | 2005 06 13<br>21 06 57 | 2739164Q.05o |
| TPCN2 | 46°10'07.25299"N<br>113°09'30.91968"W<br>1835.017 m | 2005 06 13<br>21 10 6 | 2005 06 13<br>22 23 15 | 2739164V.05o |
| DEER | 46°24'20.30241"N<br>112°44'07.90459"W<br>1363.709 m | 2005 06 13<br>15 06 32 | 2005 06 13<br>21 45 15 | DEER1641.05o |
| RES1 | 46°35'07.40556"N<br>112°54'25.51021"W<br>1267.839 m | 2005 06 13<br>15 52 33 | 2005 06 13<br>22 52 17 | RES11641.05o |
| MSOL | 46°55'45.83984"N<br>114°06'31.88621"W<br>958.450 m | 2005 06 13<br>00 00 00 | 2005 06 13<br>23 59 59 | MSOL1640.05o |
| IDNP | 45°56'22.93598"N<br>116°07'16.53030"W<br>997.103 m | 2005 06 13<br>00 00 00 | 2005 06 13<br>23 59 59 | IDNP1640.05o |

Table 6: Rover GPS receiver summary for 13 June 2005. Data rate = 1 Hz. All times are in GPS time.

| **Rover Name** | **Segments** | **Start Time** | **End Time** | **Data File** |
|---|---|---|---|---|
| NovAtel1 | I90 (Butte-Drummond)<br>SR1 (Drummond-Anaconda) | 2005 06 13<br>15 28 11 | 2005 06 13<br>19 24 39 | 00071640.05o |
| | I90 (Butte-Drummond)*<br>SR1 (Drummond-Anaconda)* | 2005 06 13<br>19 25 17 | 2005 06 14<br>01 33 51 | 00071641.05o |
| NovAtel3 | I90 (Butte-Drummond)<br>SR1 (Drummond-Anaconda) | 2005 06 13<br>15 28 15 | 2005 06 13<br>19 24 35 | 00161642.05o |
| | I90 (Butte-Drummond)*<br>SR1 (Drummond-Anaconda)* | 2005 06 13<br>19 25 26 | 2005 06 14<br>01 33 55 | 00161643.05o |
| Topcon1 | I90 (Butte-Drummond)*<br>SR1 (Drummond-Anaconda)* | 2005 06 13<br>19 24 19 | 2005 06 14<br>01 33 41 | 2629164T.05o |
| Trimble2 | I90 (Butte-Drummond)<br>SR1 (Drummond-Anaconda) | 2005 06 13<br>15 29 23 | 2005 06 13<br>19 24 22 | 39721640.05o |
| | I90 (Butte-Drummond)*<br>SR1 (Drummond-Anaconda)* | 2005 06 13<br>19 24 30 | 2005 06 14<br>01 33 35 | 39721641.05o |

* Second run with occasional stops along the way.

Table 7: Summary of INS data for 13 June 2005 survey. Data rate = 256 Hz. All times are in GPS time.

| **INS** | **Segments** | **Start Time** | **End Time** | **Data File** |
|---|---|---|---|---|
| H764G2 | SR1 (Drummond-Anaconda) | 2005 06 13<br>17 53 33 | 2005 06 13<br>19 19 59 | GM0613_2s1.SAV |
| | I90 (Butte-Drummond)*<br>SR1 (Drummond-Anaconda)* | 2005 06 13<br>19 29 34 | 2005 06 14<br>01 28 07 | GM0613_2s2.SAV |
| LN100 | I90 (Butte-Drummond)<br>SR1 (Drummond-Anaconda) | 2005 06 13<br>15 43 14 | 2005 06 13<br>19 21 07 | LN1000613s1.BIN |
| | I90 (Butte-Drummond)*<br>SR1 (Drummond-Anaconda)* | 2005 06 13<br>19 31 39 | 2005 06 13<br>22 41 17 | LN1000613s2.BIN |

* Second run with occasional stops along the way.

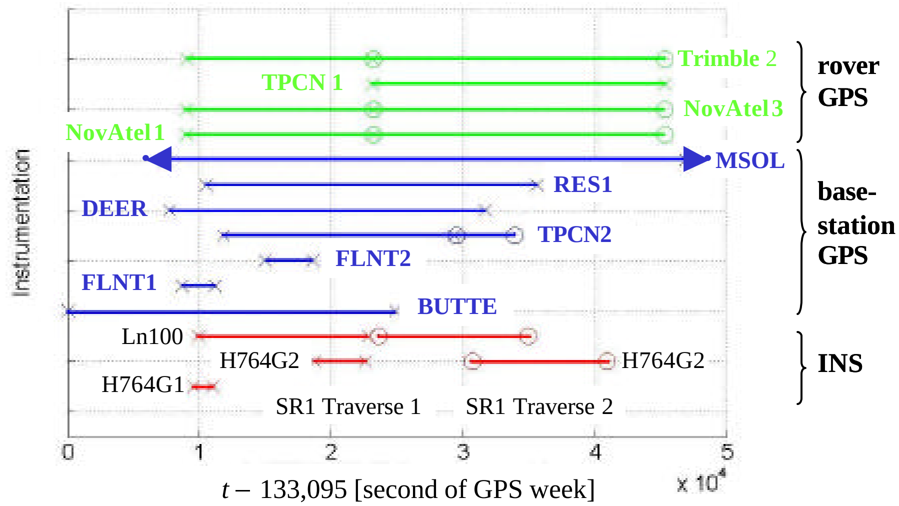



Figure 4: Comparison of time spans for each instrument on 13 June 2005.

## 1.4 Data Collection on 14 June 2005

Summaries of GPS (base stations and rover) and INS data collected on 14 June 2005 are shown in Tables 8, 9, and 10, respectively. Again, the time spans for each instrument do not coincide completely, as shown in Figure 5. The segment along I-90 from Butte to Missoula was driven continuously; while the segments southward along Route 93 and then eastward along Route 43 contained a number of stops.

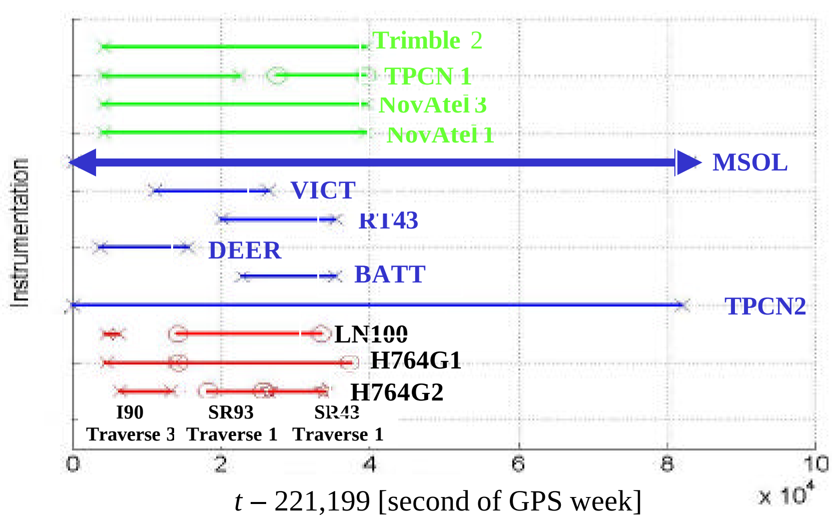



Figure 5: Comparison of time spans for each instrument on 14 June 2005.

Table 8: Summary of GPS Base Stations set up on 14 June 2005. All times are in GPS time.

| Station Name | Lat/Lon/Ht (WGS 84) | Start Time | End Time | File name |
|---|---|---|---|---|
| TPCN2 | 45°57'59.96661"N<br>112°30'48.30269"W<br>1665.735 m | 2005 06 14<br>13 26 39 | 2005 06 15<br>12 14 54 | 2739165N.05o |
| BATT | 45°38'26.75867"N<br>113°38'37.00976"W<br>1913.502 m | 2005 06 14<br>19 48 35 | 2005 06 14<br>23 15 37 | BATT1652.05o |
| DEER | 46°24'20.30278"N<br>112°44'07.90449"W<br>1363.614 m | 2005 06 14<br>14 27 34 | 2005 06 14<br>17 46 13 | DEER1651.05o |
| RT43 | 45°53'01.14267"N<br>113°07'40.28175"W<br>1746.186 m | 2005 06 14<br>19 01 33 | 2005 06 14<br>23 17 46 | RT431652.05o |
| VICT | 46°25'02.25527"N<br>114°08'49.02570"W<br>1021.924 m | 2005 06 14<br>16 32 39 | 2005 06 14<br>20 45 25 | VICT1651.05o |
| MSOL | 46°55'45.83984"N<br>114°06'31.88621"W<br>958.450 m | 2005 06 14<br>00 00 00 | 2005 06 14<br>23 59 59 | MSOL1650.05o |
| IDNP | 45°56'22.93598"N<br>116°07'16.53030"W<br>997.103 m | 2005 06 14<br>00 00 00 | 2005 06 14<br>23 59 59 | IDNP1650.05o |

Table 9: Rover GPS receiver summary for 14 June 2005. Data rate = 1 Hz. All times are in GPS time.

| Rover Name | Segments | Start Time | End Time | File name |
|---|---|---|---|---|
| NovAtel1 | I90 (Butte-Missoula)<br>SR93 (Missoula-CJP*)<br>SR43 (CJP-Big Hole) | 2005 06 14<br>14 36 39 | 2005 06 15<br>00 25 05 | 00071650.05o |
| NovAtel3 | I90 (Butte-Missoula)<br>SR93 (Missoula-CJP)<br>SR43 (CJP-Big Hole) | 2005 06 14<br>14 36 26 | 2005 06 15<br>00 25 00 | 00161650.05o |
| Topcon1 | I90 (Butte-Missoula)<br>SR93 (Missoula-CJP)<br>SR43 (CJP-Big Hole) | 2005 06 14<br>14 36 09 | 2005 06 14<br>19 41 48 | 2629165O.05o |
| Trimble2 | I90 (Butte-Missoula)<br>SR93 (Missoula-CJP)<br>SR43 (CJP-Big Hole) | 2005 06 14<br>14 36 50 | 2005 06 15<br>00 25 00 | 39721650.05o |

* CJP = Chief Joseph Pass

Table 10: Summary of INS data for 14 June 2005 survey. Data rate = 256 Hz. All times are in GPS time.

| INS | Segments | Start Time | End Time | Data File |
|---|---|---|---|---|
| H764G1 | I90 (Butte-Missoula) | 2005 06 14<br>14 48 45 | 2005 06 14<br>17 16 51 | GM0614_1s1.SAV |
| | SR93 (Missoula-CJP)<br>SR43 (CJP-Big Hole) | 2005 06 14<br>17 25 53 | 2005 06 14<br>23 50 51 | GM0614_1s2.SAV |
| H764G2 | I90 (Butte-Missoula) | 2005 06 14<br>14 56 18 | 2005 06 14<br>17 15 51 | GM0614_2s1.SAV |
| | SR93 (Missoula-CJP)<br>SR43 (CJP-Big Hole) | 2005 06 14<br>17 25 00 | 2005 06 14<br>23 55 24 | GM0614_2s2.SAV |
| LN100 | SR93 (Missoula-CJP)<br>SR43 (CJP-Big Hole) | 2005 06 14<br>17 22 54 | 2005 06 14<br>22 46 48 | LN1000614.BIN |

## 1.5 Data Collection on 15 June 2005

Finally, for 15 June 2005, Tables 11, 12, and 13, respectively, summarize the GPS and INS data collections; and Figure 6 shows the time spans for each instrument. In this case the vehicle traveled in reverse order from the previous day, from Butte along Route 43 westward, along Route 93 northward to Missoula, and ending with a non-stop run along I-90 eastward back to Butte. The traverse along Route 93 was interrupted by a detour starting in Hamilton, running along Route 269 parallel to Route 93 until Stevensville. The INS data for H764G2 on this segment were correspondingly separated into two files with endpoints at Hamilton. LN100 data were collected only during the latter part of this segment. Similarly, GPS data for the Topcon1 receiver were divided into separate observation files. Ultimately, the segment for the gravity estimation along Route 93 was terminated already at Victor.

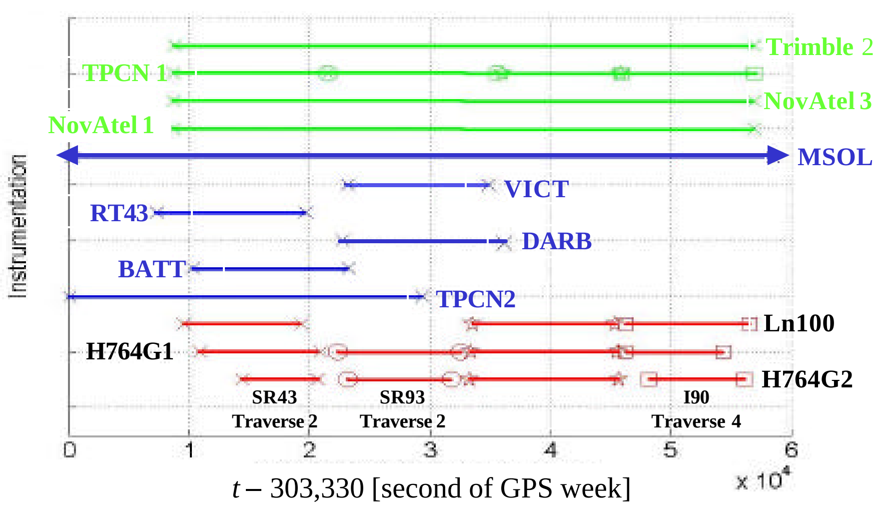



Figure 6: Comparison of time spans for each instrument on 15 June 2005.

Table 11: Summary of GPS Base Stations set up on 15 June 2005. All times are in GPS time.

| Station Name | Lat/Lon/Ht (WGS84) | Start Time | End Time | File name |
|---|---|---|---|---|
| TPCN2 | 45°57'59.96635"N<br>112°30'48.30321"W<br>1665.635 m | 2005 06 15<br>12 15 30 | 2005 06 15<br>20 24 55 | 2739166M.05o |
| BAT2 | 45°38'26.63928"N<br>113°38'36.96147"W<br>1913.415 m | 2005 06 15<br>15 07 38 | 2005 06 15<br>18 42 11 | BAT21661.05o |
| DARB | 46°01'39.66985"N<br>114°10'36.14283"W<br>1169.582 m | 2005 06 15<br>18 34 8 | 2005 06 15<br>22 07 32 | DARB1661.05o |
| 43-2 | 45°53'04.81068"N<br>113°09'12.79905"W<br>1752.202 m | 2005 06 15<br>14 16 43 | 2005 06 15<br>17 43 09 | 43-21661.05o |
| VICT | 46°25'02.25119"N<br>114°08'49.03433"W<br>1021.749 , | 2005 06 15<br>18 41 19 | 2005 06 15<br>21 56 25 | VICT1661.05o |
| MSOL | 46°55'45.83984"N<br>114°06'31.88621"W<br>958.450 m | 2005 06 15<br>00 00 00 | 2005 06 15<br>23 59 59 | MSOL1660.05o |
| IDNP | 45°56'22.93598"N<br>116°07'16.53030"W<br>997.103 m | 2005 06 15<br>00 00 00 | 2005 06 15<br>23 59 59 | IDNP1660.05o |

Table 12: Rover GPS receiver summary for 15 June 2005. Data rate = 1 Hz. All times are in GPS time.

| Rover Name | Segments | Start Time | End Time | File name |
|---|---|---|---|---|
| NovAtel1 | SR43 (Big Hole-CJP)<br>SR93 (CJP-Missoula)<br>I90 (Missoula-Butte) | 2005 06 15<br>14 41 21 | 2005 06 16<br>04 04 37 | 00071660.05o |
| NovAtel3 | SR43 (Big Hole-CJP)<br>SR93 (CJP-Missoula)<br>I90 (Missoula-Butte) | 2005 06 15<br>14 40 55 | 2005 06 16<br>04 04 31 | 00161660.05o |
| Topcon1 | SR43 (Big Hole-CJP) | 2005 06 15<br>14 40 57 | 2005 06 15<br>18 13 49 | 2629166O.05o |
| | SR93 (CJP-Missoula)<br>(only up to Hamilton) | 2005 06 15<br>18 14 23 | 2005 06 15<br>22 08 38 | 2629166S.05o |
| | Route 269 | 2005 06 15<br>22 15 53 | 2005 06 16<br>00 59 51 | 2629166W.05o |
| | I90 (Missoula-Butte) | 2005 06 16<br>01 00 11 | 2005 06 16<br>04 04 16 | 2629167B.05o |
| Trimble2 | SR43 (Big Hole-CJP)<br>SR93 (CJP-Missoula)<br>I90 (Missoula-Butte) | 2005 06 15<br>14 41 43 | 2005 06 16<br>04 04 22 | 39721660.05o |

Table 13: Summary of INS data for 15 June 2005 survey. Data rate = 256 Hz. All times are in GPS time.

| INS | Segments | Start Time | End Time | Data File |
|---|---|---|---|---|
| H764G1 | SR43 (Big Hole-CJP) | 2005 06 15 14 52 42 | 2005 06 15 18 04 25 | GM0614_1s1.SAV |
| | SR93 (CJP-Missoula) | 2005 06 15 18 27 19 | 2005 06 15 21 21 12 | GM0614_1s2.SAV |
| | | 2005 06 15 21 31 19 | 2005 06 16 00 58 02 | GM0614_1s3.SAV |
| | I90 (Missoula-Butte) | 2005 06 16 01 06 05 | 2005 06 16 03 21 05 | GM0614_1s4.SAV |
| H764G2 | SR43 (Big Hole-CJP) | 2005 06 15 14 53 36 | 2005 06 15 18 06 48 | GM0614_2s1.SAV |
| | SR93 (CJP-Missoula) | 2005 06 15 18 22 19 | 2005 06 15 21 22 25 | GM0614_2s2.SAV |
| | | 2005 06 15 21 30 19 | 2005 06 16 00 58 58 | GM0614_2s3.SAV |
| | I90 (Missoula-Butte) | 2005 06 16 01 05 18 | 2005 06 16 03 58 51 | GM0614_2s4.SAV |
| LN100 | SR43 (Big Hole-CJP) | 2005 06 15 14 54 45 | 2005 06 15 17 40 21 | LN1000615s1.BIN |
| | SR93 (CJP-Missoula) | 2005 06 15 21 34 05 | 2005 06 16 00 56 59 | LN1000615s3.BIN |
| | I90 (Missoula-Butte) | 2005 06 16 01 05 18 | 2005 06 16 03 58 51 | LN1000615s4.BIN |

## 2. Data Processing Techniques

The techniques to estimate the gravity components follow basically those developed at OSU for airborne GPS/INS vector gravimetric systems. Details of the processing and estimation procedures may be found in previous reports and papers (e.g., Kwon and Jekeli, 2001; Jekeli, 2000, ch.10; Jekeli and Li, 2004). The GPS data from each pair of base station and rover receivers were processed using the software Applanix™ that also predicts standard deviations for the position solutions. The latter provided some indication of where GPS-derived positions might be adversely affected by poor satellite geometry or poor resolution of phase cycle ambiguities. We chose to use this software exclusively since previous experience (Jekeli and Li, 2005) showed that, among those that we currently have available (Applanix™, Trimble Geomatics Office, NGS's KARS, MIT's GAMIT/Track), it yields the best and consistently most reliable solution. Our analyses with the present data also revealed a correlation between relatively poor GPS standard deviations and degraded gravity estimates (see Section 3). Thus, from the various solutions implied by different pairs of rover/base-station receivers, we chose that which seemed to offer the least variation in standard deviation. To repair gaps in the GPS solution we applied a simple linear interpolation.

We did not incorporate the lever-arm effect caused by the offset of the GPS antenna from the INS, since we do not expect the vehicle to rotate significantly. However, the insignificance of this effect requires further verification. The basic registration for all GPS and INS data is the GPS time. To convert this to along-track registration in terms of distance, we used a simple algorithm that adds a differential linear element sequentially to a defined starting point for each segment. The differential line element was determined from the best GPS solution available. This one-to-one relationship between GPS time and along-track distance was used for all displays of our results as well as for interpolation purposes when comparing two (possibly oppositely run) traverses along the same road.

On occasion, small diversions off the main road were included in the actual traverse in order to visit DOV and gravity benchmarks. However, it was decided to exclude these from the gravity estimation by interpolating the *estimates* across the detour. For the present analysis this does not adversely impact the results and also does not omit potentially useful data. The resolution of the estimation is not sufficient to yield a level of detail suggested by these excursions. Figure 7 provides two examples of an extraneous loop that was removed from the analysis of the gravity estimation.

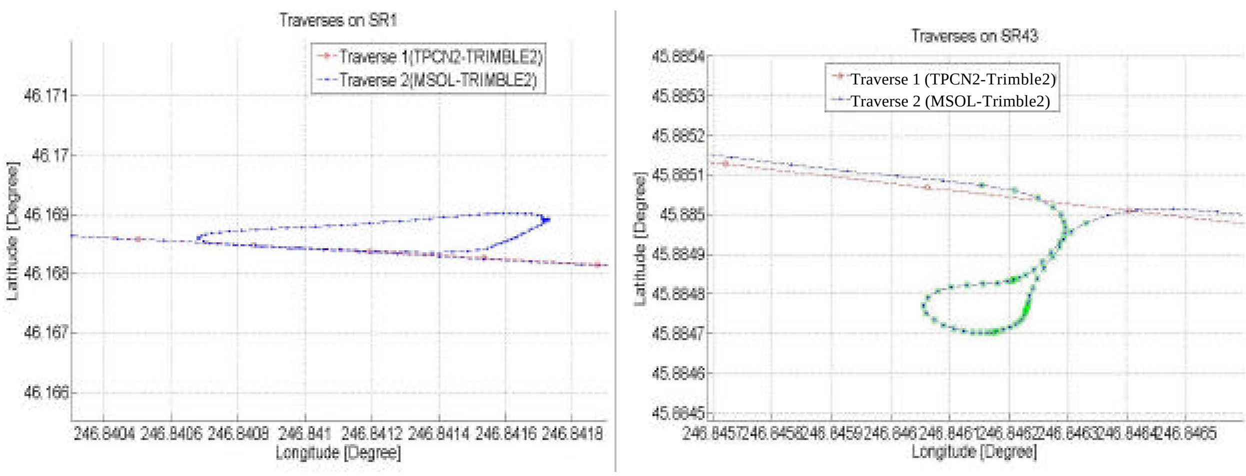



Figure 7: Examples of small excursions of the GPSVan to visit a DOV point along Route 1 (left) and a base station point along Route 43 (right).

With a fit of B-splines to the GPS-derived positions, GPS accelerations were determined by analytic differentiation. Inherent in this scheme is a smoothing process that filters noise with temporal frequencies higher than some specified value. Based on several test computations, we used a 180-second filter. Next, the IMU data (delta velocities and delta angles) were combined to obtain accelerations in the GPS coordinate frame, and subsequently smoothed at the same level as the GPS accelerations. The difference between the GPS and IMU accelerations is the gravitational acceleration (the lever-arm effect was not considered; see above). Instead of a simple subtraction, we determined these differences in a Kalman filter that attempts to remove some systematic errors associated with the inertial sensors (biases and scale factor errors in the accelerometers and gyros). These algorithms require an initial set of positions and orientations of the system, which were taken from the GPS and INS navigation solutions. Despite these various filters and initial data, the estimates of the gravity component include unknown biases and possible trends that can be solved only with external gravity and DOV control data (Serpas and Jekeli, 2005). In this initial processing, these biases (and trends) were not determined.

## 3. Gravity Vector Estimation Results

The main objective of this analysis is to provide an initial assessment of the ability of the ground vehicle surveys to yield along-track estimates of all three components of the gravity disturbance vector. Although endowed with a substantial amount of vertical gravity control data, the survey area contained only a few control points for the horizontal components in terms of direct independent measurements. Two sources of indirect horizontal gravity control could be considered in evaluating our surveys. One is the DEFLEC99 data set obtained by NGS from their Geoid99 model; the other is our own prediction of the deflections of the vertical using the program GEOCOL (least-squares collocation software; Tscherning, 2005) applied to the NGA gravity data in the survey area, as well as the EGM96 reference model. Either one, however, may include unknown model errors and may not have sufficiently high accuracy in the maximum resolution to evaluate our estimates, which have a theoretical resolution of about 2 km (equivalent to that of DEFLEC99 and the NGA gravity data). Comparing the horizontal gravity disturbances derived from the NGS geoid model (DEFLEC99) to those obtained through least-squares collocation (GEOCOL) shows agreement only of the order of ten mgal (or more), as shown in Table 14. Figure 8 displays profiles of these horizontal component estimates along SR43.

Table 14: Comparison of horizontal gravity disturbances from NGS (DEFLEC99) and NGA gravity data using GEOCOL along specific road segments (for a definition, see below).

| | Mean (mgal) | STD (mgal) |
|---|---|---|
| North component, SR1 | -4.14 | 11.93 |
| East component, SR1 | 41.56 | 31.97 |
| North component, SR43 | -11.82 | 7.25 |
| East component, SR43 | 14.54 | 12.22 |

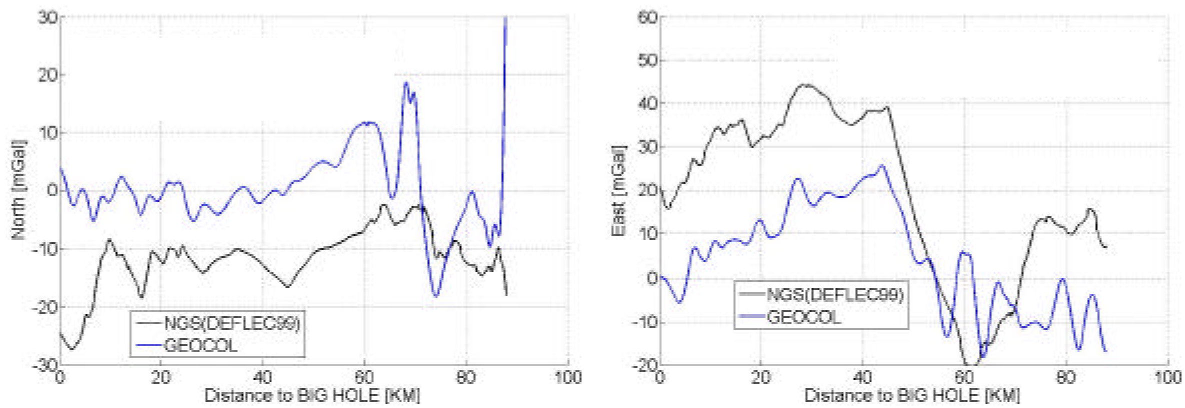



Figure 8: Comparison of horizontal gravity disturbance components from DEFLECT99 (based on Geoid99) and from GEOCOL (estimated form regional gravity data and EGM96).

Aside from comparison to control data, repeatability is another form of assessing the precision of the estimated gravity components. All segments of the survey were repeated at least once under different circumstances. Traverses on I-90 were separated by one or two days, and even by 1.5 months, and were run in opposing directions. Repeat traverses along Routes 93 and 43 were conducted in reverse directions and on different days. The repeat traverse on Route 1 was run on the same day and in the same direction but with more frequent stops along the second run. Systematic errors in the data would likely not repeat exactly unless they were strongly correlated with the geography of the traverse (which is a remote possibility). Assuming that they do not repeat in the same way (i.e., they are random from one sortie to the next), any commonality in the estimates along repeated routes must be due to gravity.

Since repeatability was considered of primary value in our assessment, the total set of traverses was divided into four principal segments: I90, along Interstate Route 90 between Butte and Missoula; SR1, along Route 1 between Drummond and Anaconda; SR93, along Route 93 between Missoula and Chief Joseph Pass; and SR43, along Route 43 between Chief Joseph Pass and Big Hole River base station (intersection of Routes 43 and 569). With this organization, we defer to a later time the analysis of some data collected along the eastern part of Route 43, along route 569, and west of Missoula, along Route 12 (see Figure 1). Table 15 provides the description of the segments in terms of geographical endpoints; and Table 16 defines individual traverses in terms of GPS time for each endpoint.

Generally, H764G2 was the only INS that yielded consistently acceptable results. For those times that H764G1 and LN100 generated data they were often of significantly poorer quality and we do not include them in the main analysis (see Appendix).

Table 15: Definition of Segments in terms of geographical coordinates for the endpoints.

| Segment | Start Point | WGS84 (*x*,*y*,*z*) Coordinates of Start Point [m] | End Point | WGS84 (*x*,*y*,*z*) Coordinates of End Point [m] | Total Length [km] |
|---|---|---|---|---|---|
| SRI | Drummond | -1725890.670<br>-4033636.577<br>4615588.544 | Anaconda | -1723167.736<br>-4080271.057<br>4576175.272 | 90.125 |
| SR43 | Big Hole | -1746508.632<br>-4091671.161<br>4557511.752 | Chief Joseph Pass | -1812151.977<br>-4079851.857<br>4543086.315 | 88.731 |
| SR93 | Chief Joseph Pass | -1812125.681<br>-4079863.528<br>4543085.856 | Victor | -1796417.573<br>-4013276.672<br>4606175.537 | 106.828 |
| I90 | Missoula | -1775630.312<br>-3992022.744<br>4632411.855 | Butte | -1700004.188<br>-4102480.270<br>4565188.418 | 191.364 |

Table 16: Definition of traverses in terms of GPS time for each endpoint.

| Segment / Traverse | Date | Endpoint and GPS Time [second of GPS week] | Endpoint and GPS Time [second of GPS week] |
|---|---|---|---|
| SR1, Traverse 1 | 13 June 2005 | Drummond, 151815 | Anaconda, 155644 |
| SR1, Traverse 2 | 13 June 2005 | Drummond, 163825 | Anaconda, 173982 |
| SR43, Traverse 1 | 14 June 2005 | CJP, 247686 | Big Hole, 254964 |
| SR43, Traverse 2 | 15 June 2005 | Big Hole, 317718 | CJP, 323977 |
| SR93, Traverse 1 | 14 June 2005 | Victor, 239395 | CJP, 246800 |
| SR93, Traverse 2 | 15 June 2005 | CJP, 326494 | Victor, 335139 |
| I90, Traverse 1 | 28 April 2005 | Butte, 408496 | Missoula, 414494 |
| I90, Traverse 2 | 28 April 2005 | Missoula, 418846 | Butte, 423681 |
| I90, Traverse 3 | 14 June 2005 | Butte, 227601 | Missoula, 234476 |
| I90, Traverse 4 | 15 June 2005 | Missoula, 351509 | Butte, 359419 |

### 3.1 SR1 Analysis

Figures 9 and 10 show the standard deviations of the DGPS positions predicted by Applanix™ for the two traverses along the Route 1 segment (SR1), about 90 km in length, running between Drummond and Anaconda (see also Figure 2). Both traverses were run on 13 June 2005 and in the same direction, from Drummond to Anaconda. Figure 11 shows two sets of standard deviations (BUTTE-Trimble2 and TPCN2-Trimble2) for the first traverse in terms of along-track distance from the endpoint, Drummond. The relatively large standard deviations near the 56 km point from Drummond correspond to the GPS time of about 154,000 [s, GPS week]. GPS standard deviations for the second traverse in terms of distance from the Drummond endpoint are shown in Figure 12 for the pairs MSOL-Trimble2 and MSOL-NovAtel1. This traverse included several stops, which, however, do not correlate with the larger standard deviations.

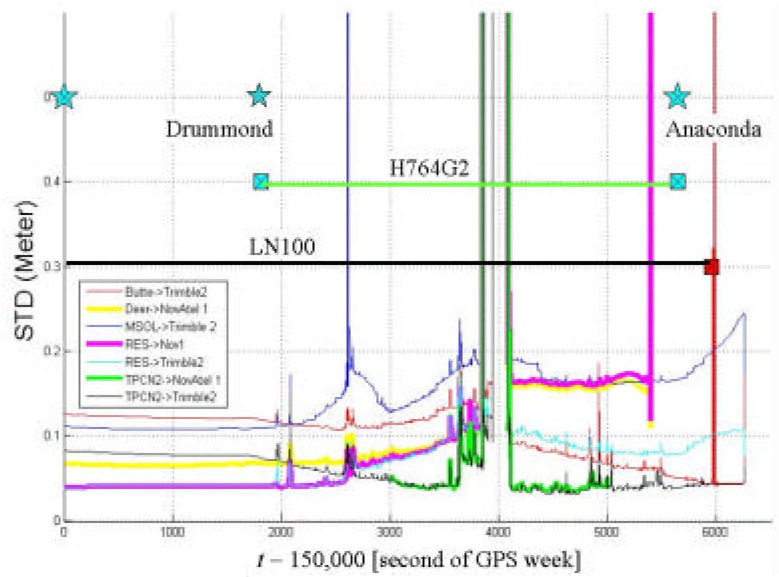



Figure 9: Predicted standard deviations of DGPS solutions for the GPSVan, Traverse 1 of Segment SR1, according to Applanix™ software and for different receiver/base station combinations.

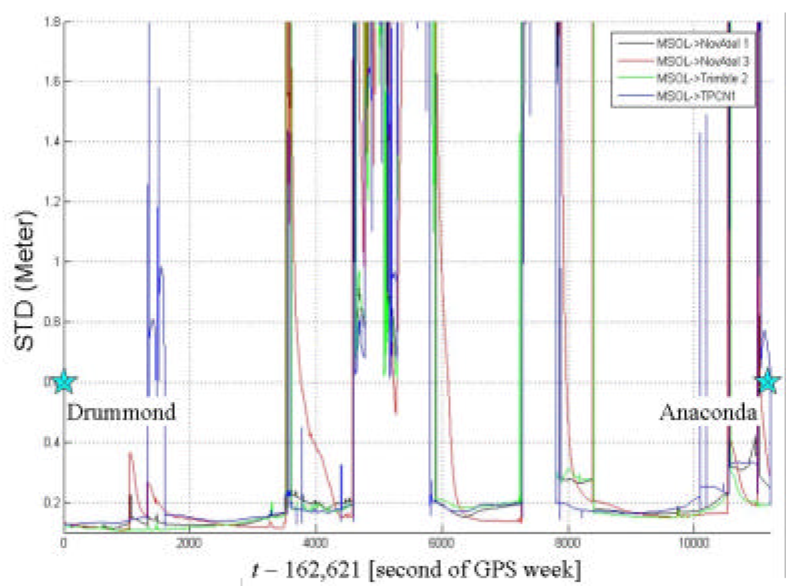



Figure 10: Predicted standard deviations of DGPS solutions for the GPSVan, Traverse 2 of segment SR1, according to Applanix™ software and for different receiver/base station combinations. H764G2 and LN100 data were collected for the entire indicated interval.

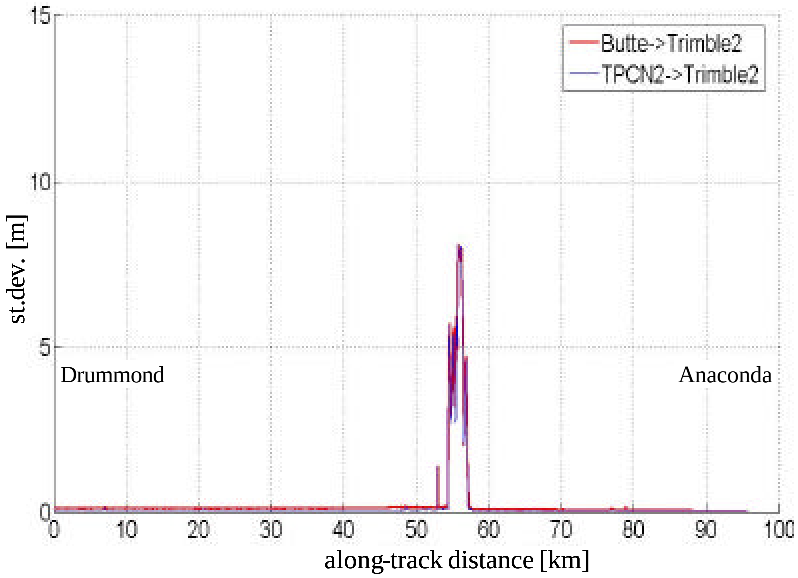


Figure 11: Same as Figure 9, but for only two rover/base-station pairs and with respect to along-track distance from Drummond.

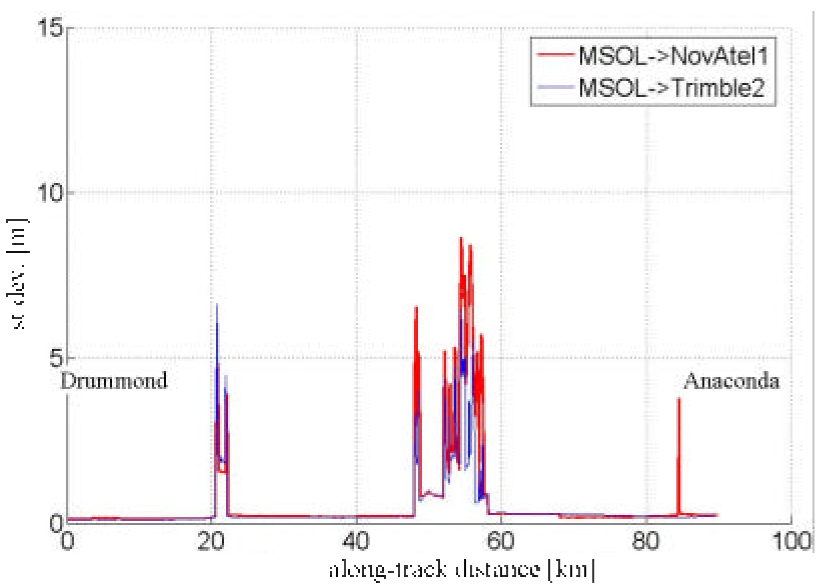


Figure 12: Same as Figure 10, but for only two rover/base-station pairs and with respect to along-track distance from Drummond.

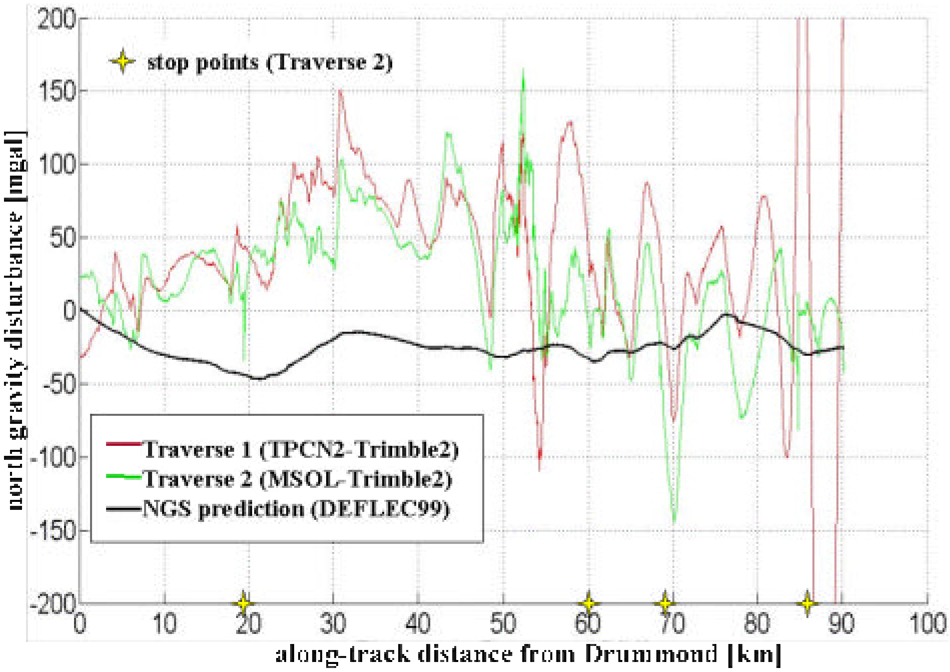
Table 17 identifies four solutions with H764G2 for the two traverses and different GPS receiver pairs. Figures 13(abc) compare the two solutions, SR1-1 and SR1-3, along Traverses 1 and 2, respectively, for the three gravity disturbance components. There is general agreement for the down component (Figure 13c) on both traverses and with respect to the values interpolated from the NGA gravity control points, except for a bias. Horizontal components do not agree with the DEFLEC99 (nor the GEOCOL) derived values, except at the very long wavelengths (there is also an unsolved bias). However, it is evident that there is a strong correlation in the high frequency constituents between the two traverses. Arguably, the source of these correlated parts are gravity disturbances, because it is unlikely that the same systematic errors are repeated exactly along the two traverses. There is the remote possibility that the road conditions impact systematic errors in a predictable way, but similar types of correlation are obtained for other segments where the traverses were run in opposite directions (different sides of the road or divided highway). Thus, it seems very unlikely that these correlations in the estimates are systematic errors induced by road conditions.

It is also clear without much analysis that there are significant errors in the estimates, particularly between the 50 and 60 km points, exactly where the GPS position solution is associated with large standard deviation (Figures 11 and 12). Figure 12 also shows some GPS problems for Traverse 2 near the 22 km point and especially between the 60 km and 70 km points. However, evidence of error in all three gravity disturbance components in this case occurs rather at the points associated with stop points (19 km, 60 km, 69 km, 86 km). Thus while GPS inaccuracy over longer intervals certainly has a detrimental effect on the estimation, the stop points also cause some kind of discontinuity or general degradation in the estimates.

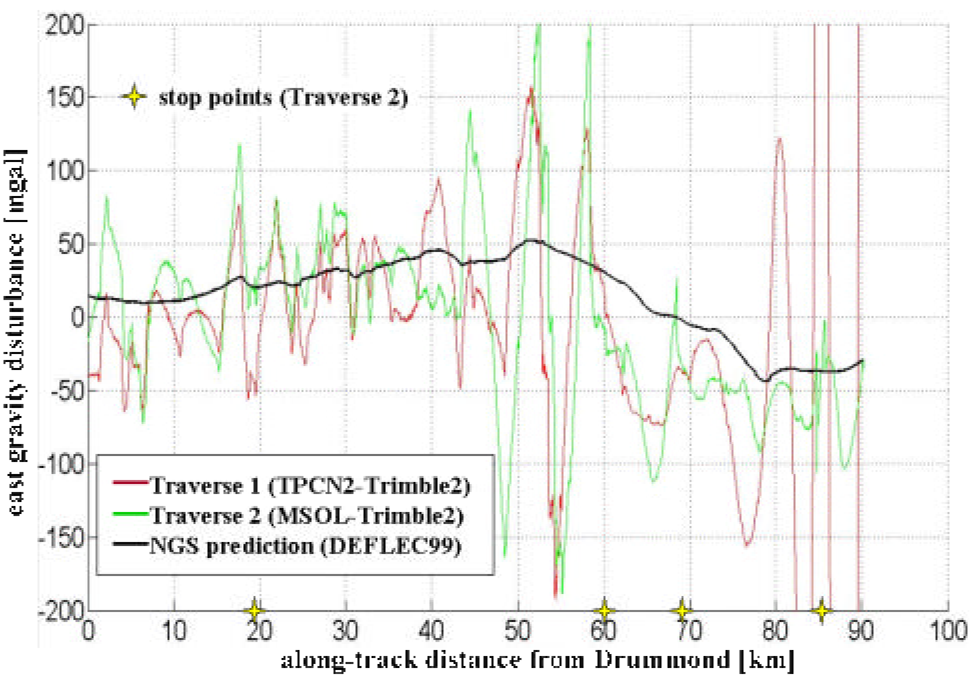
The same general qualitative conclusion may be drawn by comparing the solutions SR1-2 and SR1-4 (Figures 14abc), which use a different set of rover/base-station GPS receivers. The essential difference with respect to the previous comparison is due to large irreparable gaps in the MSOL-NovAtel1 GPS solution that yielded completely erroneous gravity estimates after the 50 km point. (The gap is not evident in Figure 12 since it occurred in one of the excursion loops, but the data processing is done continuously through all loops, which are eliminated only in the final estimates.) This, again, points to the requirement for consistently precise GPS solutions.

Table 17: Instrumentation used in the different solutions for the Segment SR1.

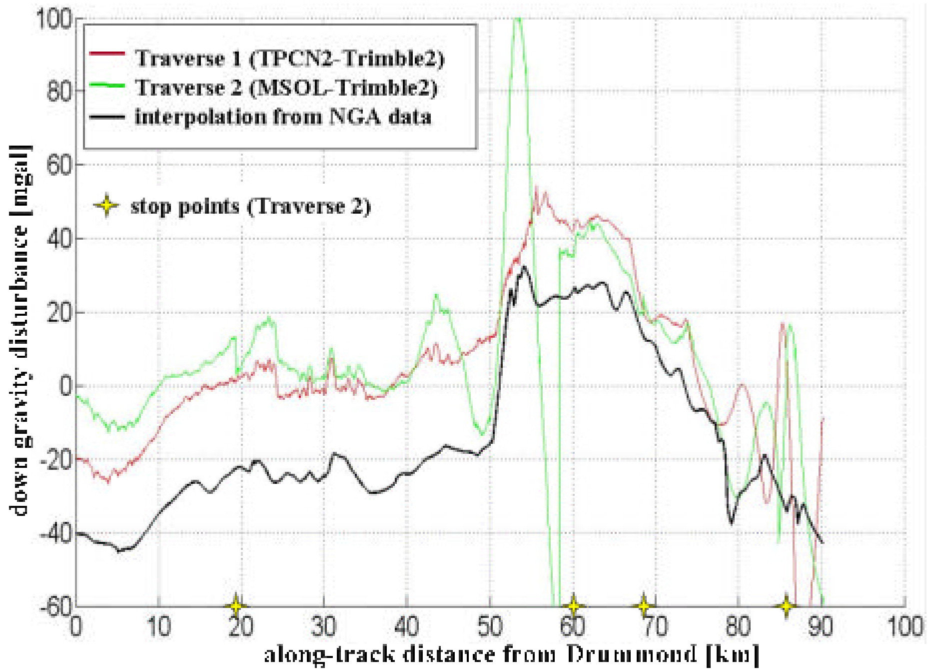
| Solution Name | Traverse | Date | INS | Rover Receiver | Base Station |
|---|---|---|---|---|---|
| SR1-1 | 1 | 13 June | H764G2 | Trimble2 | TPCN2 |
| SR1-2 | 1 | 13 June | H764G2 | Trimble2 | BUTTE |
| SR1-3 | 2 | 13 June | H764G2 | Trimble2 | MSOL |
| SR1-4 | 2 | 13 June | H764G2 | NovAtel1 | MSOL |


stop points (Traverse 2)
north gravity disturbance [mgal]
Traverse 1 (TPCN2-Trimble2)
Traverse 2 (MSOL-Trimble2)
NGS prediction (DEFLEC99)
along-track distance from Drummond [km]

a)

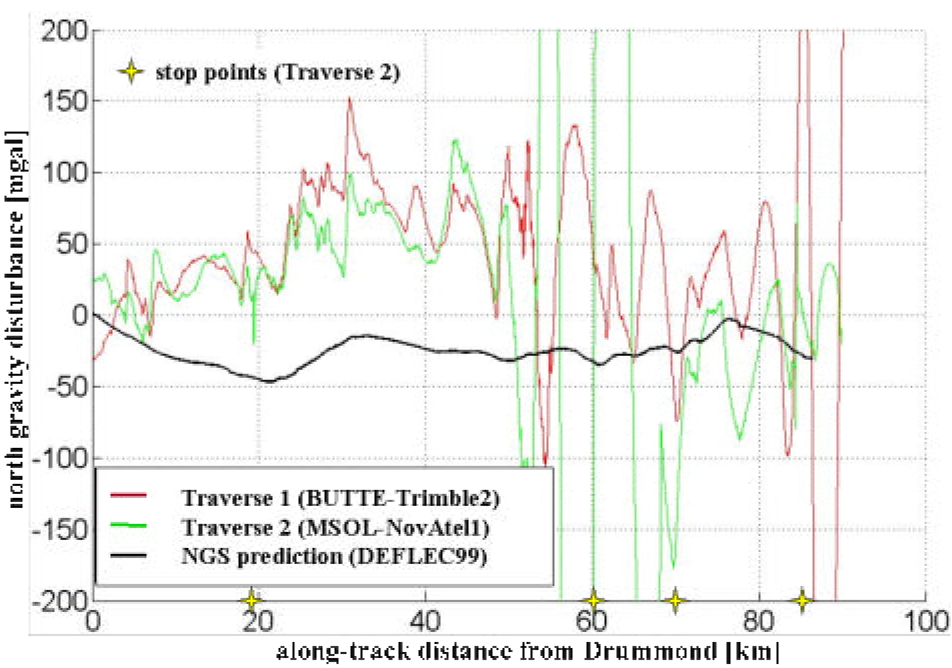

stop points (Traverse 2)
east gravity disturbance [mgal]
Traverse 1 (TPCN2-Trimble2)
Traverse 2 (MSOL-Trimble2)
NGS prediction (DEFLEC99)
along-track distance from Drummond [km]

b)

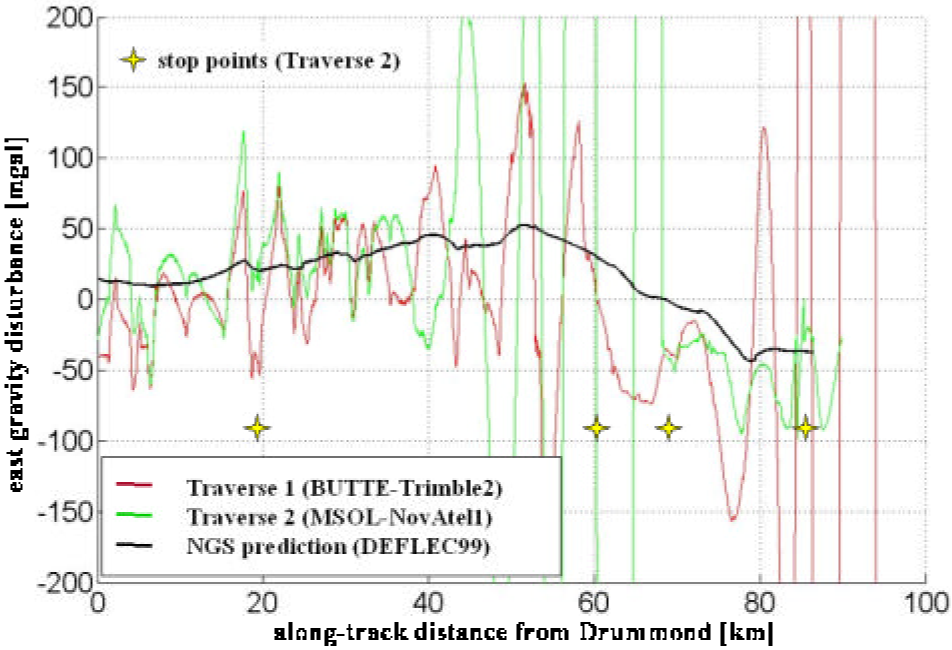



c)

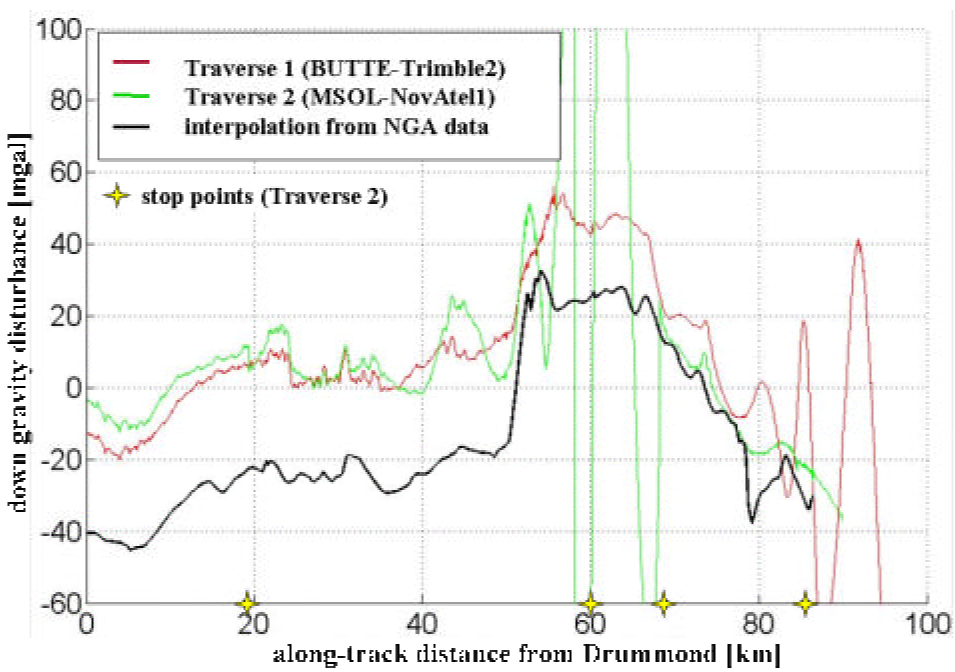
Figure 13: Gravity vector estimates along SR1, from Drummond to Anaconda, MT, using data collected on 13 June 2005 (Traverses 1 and 2): a) north component, b) east component, c) down component. "Control" data, stop points for Traverse 2, and rover/base-station receivers are also indicated.




a)




b)




c)

Figure 14: Gravity vector estimates along SR1, from Drummond to Anaconda, MT, using data collected on 13 June 2005 (Traverses 1 and 2): a) north component, b) east component, c) down component. "Control" data, stop points for Traverse 2, are also indicated. These figures differ from Figures 13abc in the selection of rover/base-station GPS receivers.

### 3.2 SR93 and SR43 Analyses

The segments along Routes 93 and 43 were traversed twice each, and a qualitative analysis may be attempted as for Segment SR1 (Section 3.1). Segment SR93 runs along Route 93 between Victor and Chief Joseph Pass (CJP); while, Segment SR43 continues from Chief Joseph Pass to Big Hole (along Route 43 to the intersection with Route 569). Traverse 1, on 14 June 2005, begins in Victor, runs through CJP and ends at Big Hole. Traverse 2, on 15 June 2005, runs in the opposite direction, from Big Hole through CJP to Victor. A number of stops were made on both traverses, as indicated in Figure 2. Figure 15 shows the standard deviations as predicted by the Applanix™ software for Traverse 1 (14 June); Figures 16 and 17 provide standard deviations as functions of along-track distance for each segment. Likewise, Figures 18 and 19 show corresponding standard deviations for the segments of Traverse 2 (15 June). Although all rover/base-station receiver pairs yield essentially equivalent results, we selected TPCN2-Trimble2 for the analysis of Traverse 1 (14 June) and MSOL-Trimble2 for Traverse 2 (15 June). Table 18 characterizes the different solutions in terms of instruments and base stations used to estimate the gravity vector.

Table 18: Instrumentation used in the different solutions for the Segments SR43 and SR93.

| Solution Name | Traverse | Date Run | INS | GPS (rover) | Base Station |
|---|---|---|---|---|---|
| SR93-1 | 1 | 14 June | H764G2 | Trimble 2 | TPCN2 |
| SR93-2 | 2 | 15 June | H764G2 | Trimble 2 | MSOL |
| SR43-1 | 1 | 14 June | H764G2 | Trimble 2 | TPCN2 |
| SR43-2 | 2 | 15 June | H764G2 | Trimble 2 | MSOL |

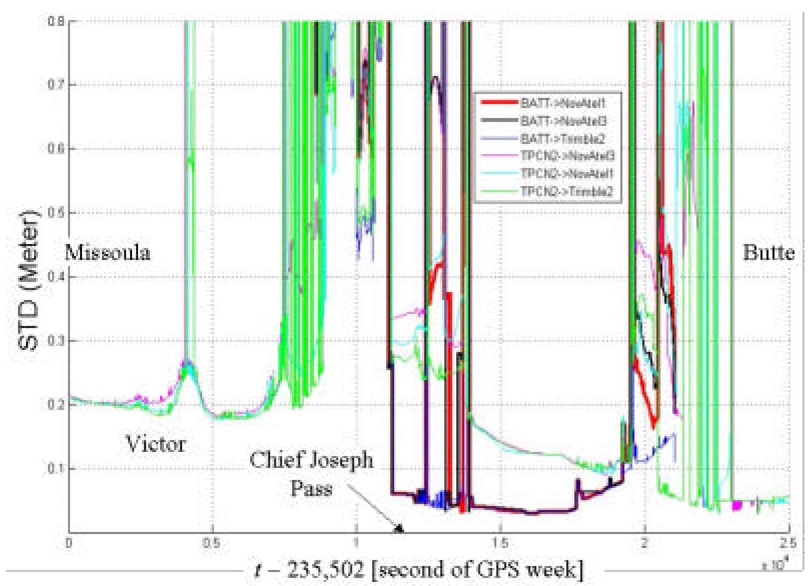



Figure 15: Predicted standard deviations of DGPS solutions for the GPSVan, Traverse 1 (14 June 2005) of segments SR93 and SR43, according to Applanix™ software and for different receiver/base station combinations.

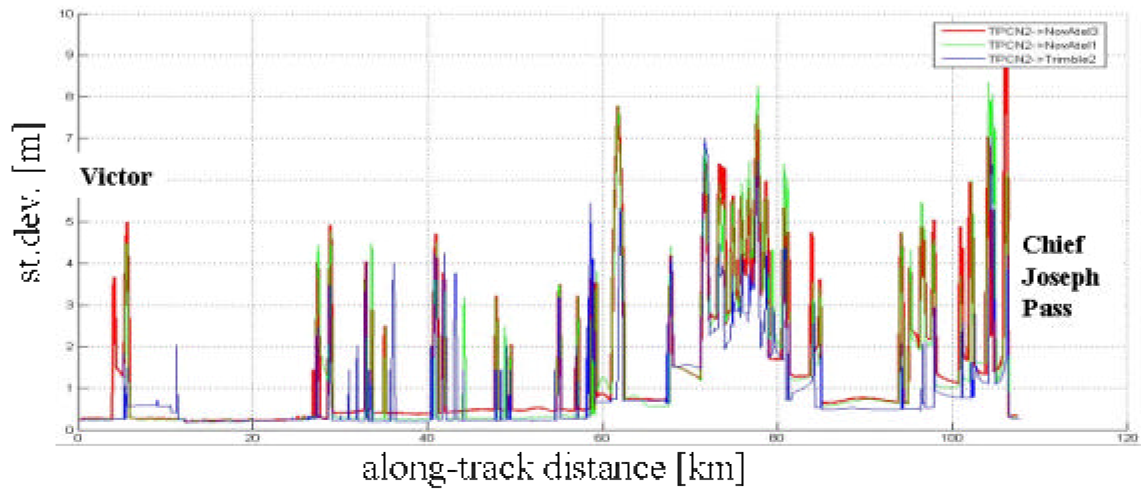



Figure 16: Same as Figure 15, but for Segment SR93 only and just three rover/base-station pairs, with respect to along-track distance from Victor.

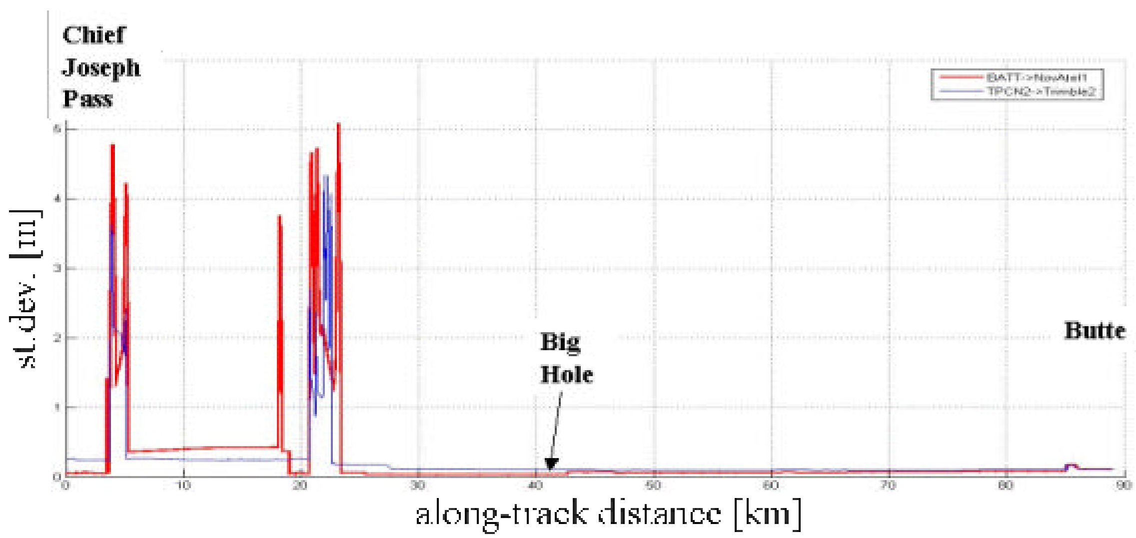



Figure 17: Same as Figure 15, but for Segment SR43 only and just two rover/base-station pairs, with respect to along-track distance from Chief Joseph Pass.

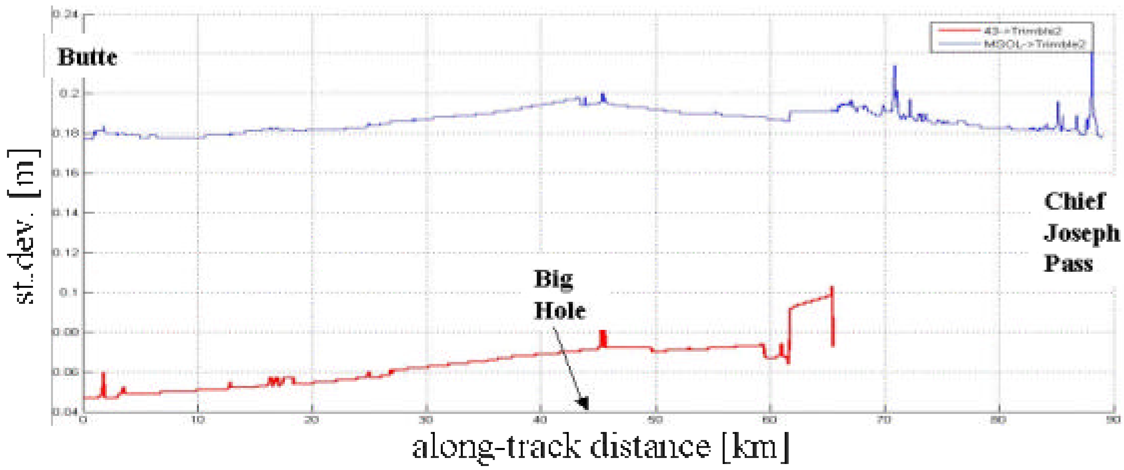



Figure 18: Predicted standard deviation of the GPS solution of Traverse 2 (15 June) for Segment SR43 only and two rover/base-station pairs, with respect to along-track distance from Butte.

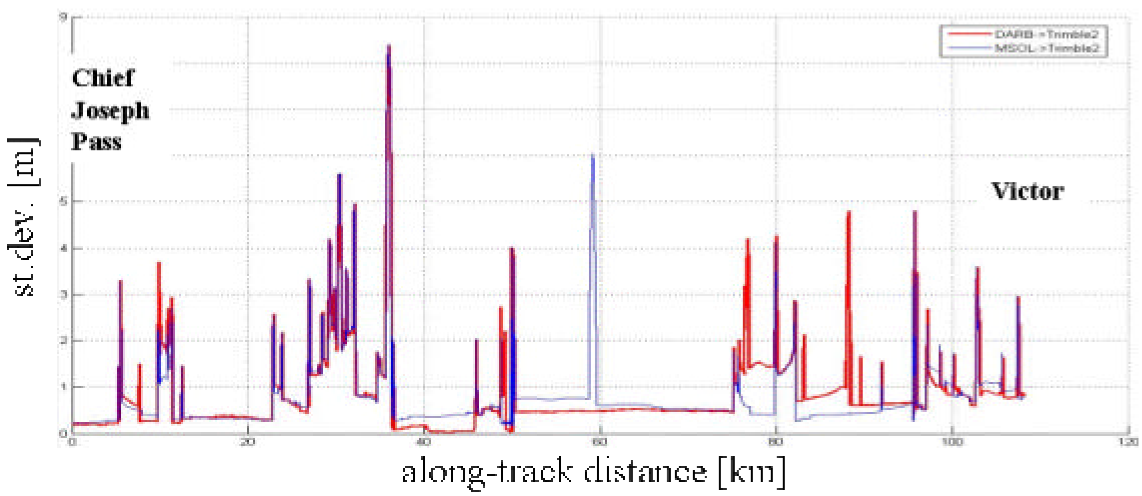
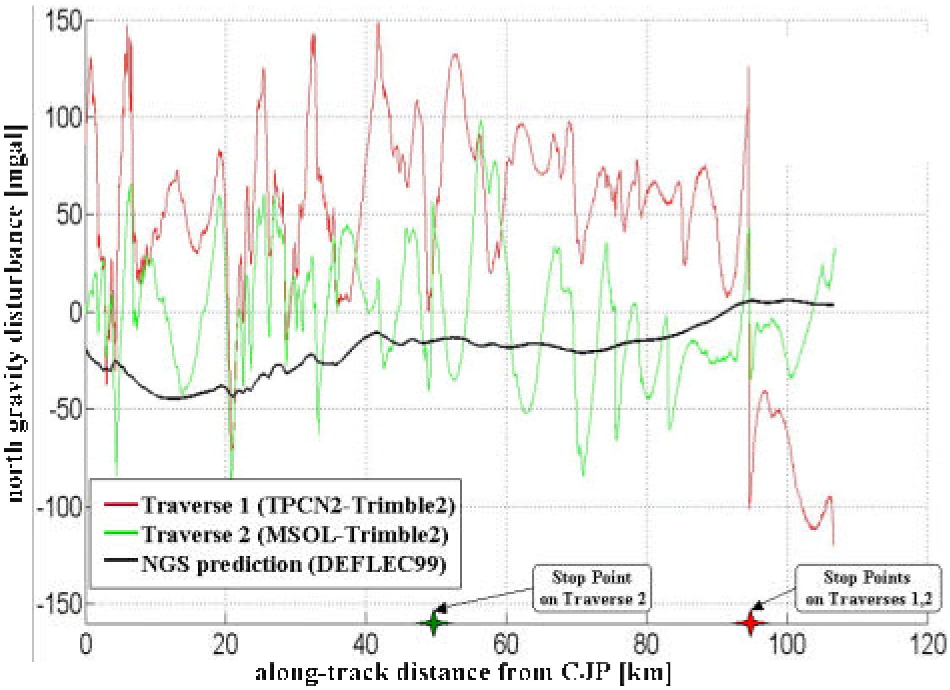


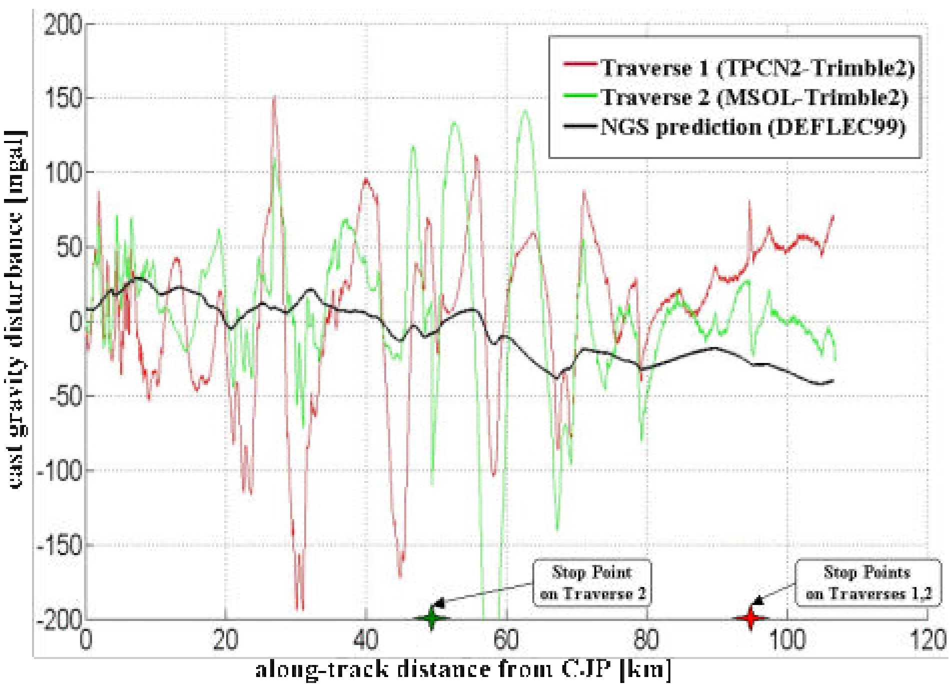

Figure 19: Predicted standard deviation of the GPS solution of Traverse 2 (15 June) for Segment SR93 only and two rover/base-station pairs, with respect to along-track distance from Chief Joseph Pass.

Gravity disturbance estimates are shown for Segments SR93 and SR43, respectively, in Figures 20(abc) and Figures 21(abc). There appears to be little correlation between the two traverses along Segment SR93 for all components, even the down component. This is likely due the relatively poor GPS position solutions as indicated by the rather frequent large predicted standard deviations as shown in Figures 16 and 19. The results for this segment are thus deemed unsuccessful. Segment SR43, on the other hand, shows remarkable consistency in the down gravity disturbance component, especially at the higher frequencies, even more between traverses than with respect to the interpolated values from the control data. The horizontal components also exhibit significant correlation at the higher frequencies between the two traverses, as in the case of Segment SR1.



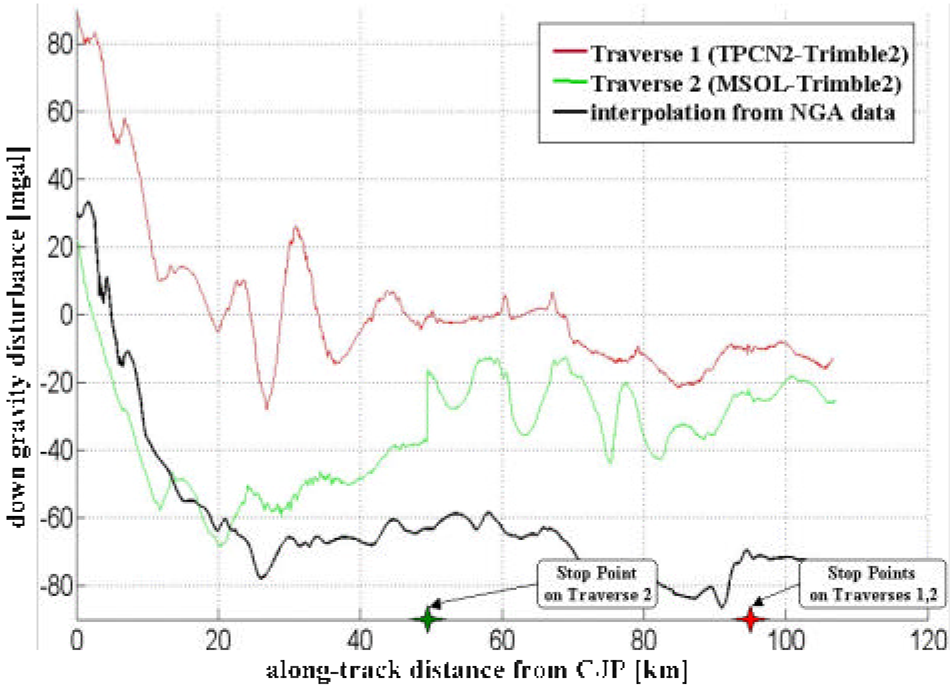
150
100
50
0
-50
-100
-150
north gravity disturbance [mgal]
Traverse 1 (TPCN2-Trimble2)
Traverse 2 (MSOL-Trimble2)
NGS prediction (DEFLEC99)
Stop Point on Traverse 2
Stop Points on Traverses 1,2
0
20
40
60
80
100
120
along-track distance from CJP [km]


a)



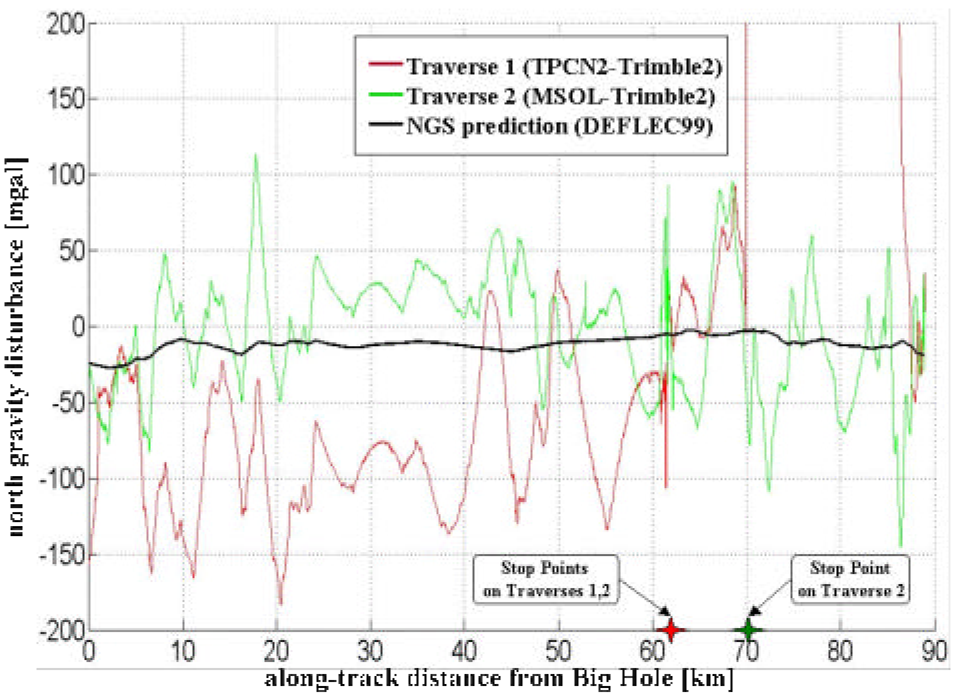
200
150
100
50
0
-50
-100
-150
-200
east gravity disturbance [mgal]
Traverse 1 (TPCN2-Trimble2)
Traverse 2 (MSOL-Trimble2)
NGS prediction (DEFLEC99)
Stop Point on Traverse 2
Stop Points on Traverses 1,2
0
20
40
60
80
100
120
along-track distance from CJP [km]


b)

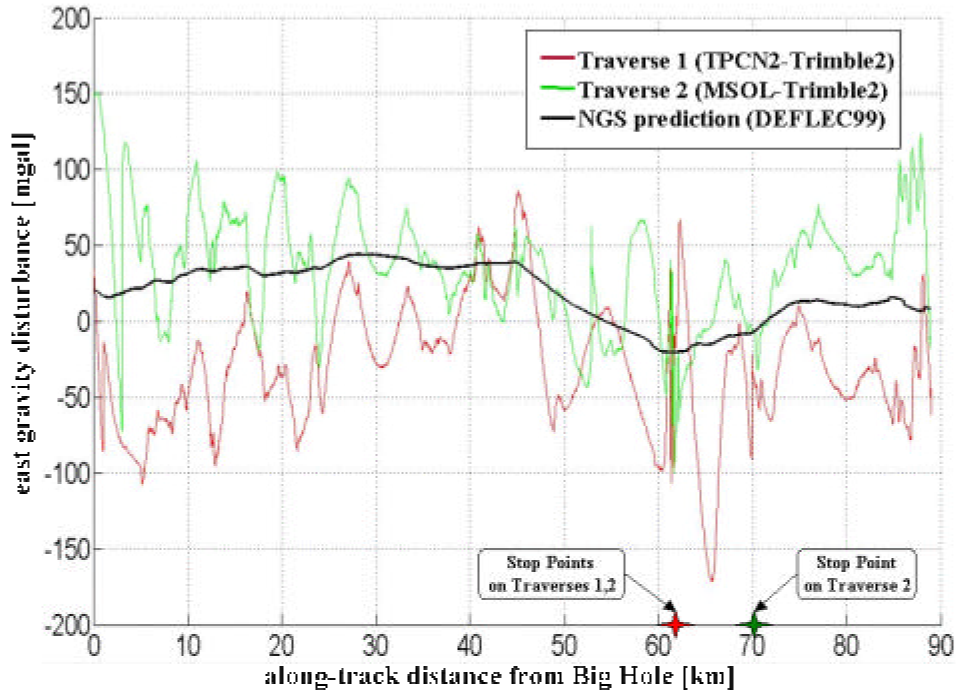



c)

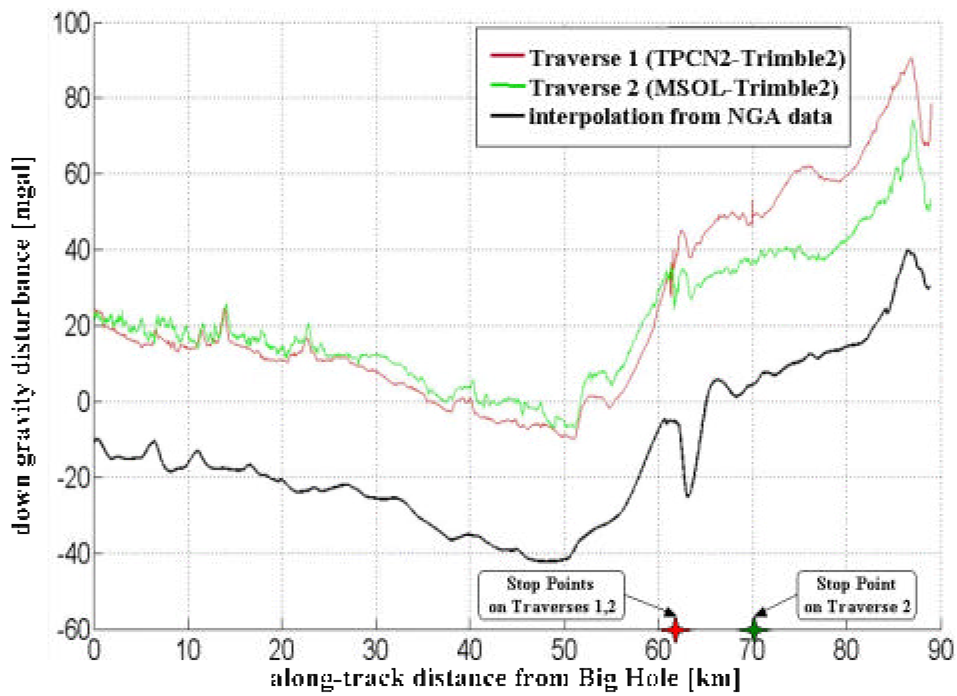
Figure 20: Gravity vector estimates along SR93, from CJP to Victor, using data collected on 14-15 June 2005 (Traverses 1 and 2, respectively): a) north component, b) east component, c) down component. "Control" data, stop points, and base-station/rover receivers are also indicated.




a)




b)




c)

Figure 21: Gravity vector estimates along SR43, from Big Hole to Chief Joseph Pass (CJP), using data collected on 14-15 June 2005 (Traverses 1 and 2, respectively): a) north component, b) east component, c) down component. “Control” data, stop points, and base-station/rover receivers are also indicated.

### 3.3 I90 Analysis

The segment along I-90, about 191 km in length, was traversed four times on three days (28 April, 14-15 June 2005), with two partial runs on one day (13 June 2005). Traverse 1, on 28 April, began in Butte and ended in Missoula; with Traverse 2 being the reverse on the same day. Traverse 3 was the same as Traverse 1, but on 14 June; and Traverse 4, on the next day, 15 June, was the reverse of Traverse 3. The partial traverse on 13 June during the second run along I-90 (for which we have INS data) is also included, as Traverse 5. Table 19 characterizes the corresponding solutions that contribute to the main analysis. Recall that potentially suitable INS data for Traverse 1 were generated only by H764G1 and LN100, but neither of these instruments performed well and results are relegated to the Appendix. Figure 22 shows the standard deviations corresponding to the GPS solutions obtained for the 28 April traverses. Clearly, the NovAtel2 receiver consistently offered the lowest values, indicating that it likely represents the best DGPS solution. Standard deviations for the GPS solutions of Traverses 3 and 4 on 14-15 June are shown with respect to along-track distance in Figure 23. Even though different base stations were used for these two traverses, the standard deviations almost mirror each other, indicating that they are geographically correlated, possibly due to GPS outages associated with overpasses or other obstructions.

Table 19: Instrumentation used in the different solutions for the Segment I90.

| Solution Name | Traverse | Date | INS | GPS (rover) | Base Station |
|---|---|---|---|---|---|
| I90-2 | 2 | 28 Apr 05 | H764G2 | NovAtel 2 | MSOL |
| I90-3 | 3 | 14 Jun 05 | H764G2 | Trimble 2 | TPCN2 |
| I90-4 | 4 | 15 Jun 05 | H764G2 | Trimble 2 | MSOL |
| I90-5* | 5 | 13 Jun 05 | H764G2 | Trimble 2 | MSOL |

* Part of total segment

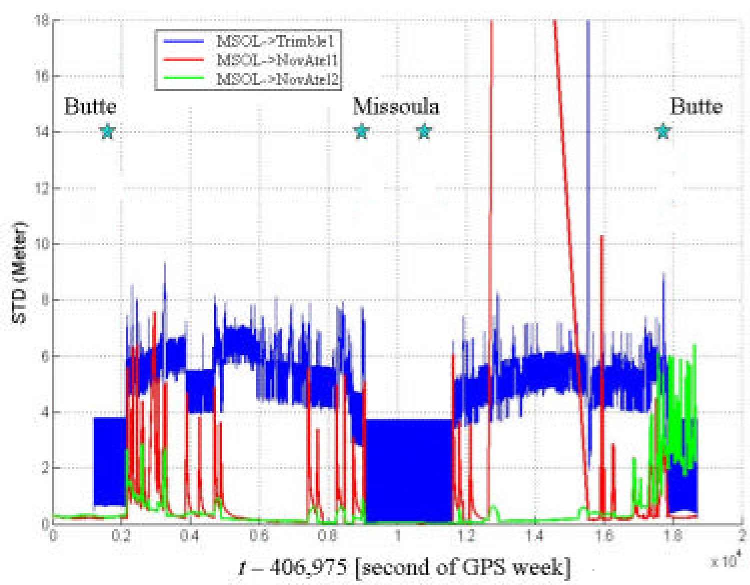



Figure 22: Predicted standard deviations of DGPS solutions for the GPSVan according to Applanix™ software and for different rover receivers on Traverses 1 and 2 along Segment I90, 28 April 2005.

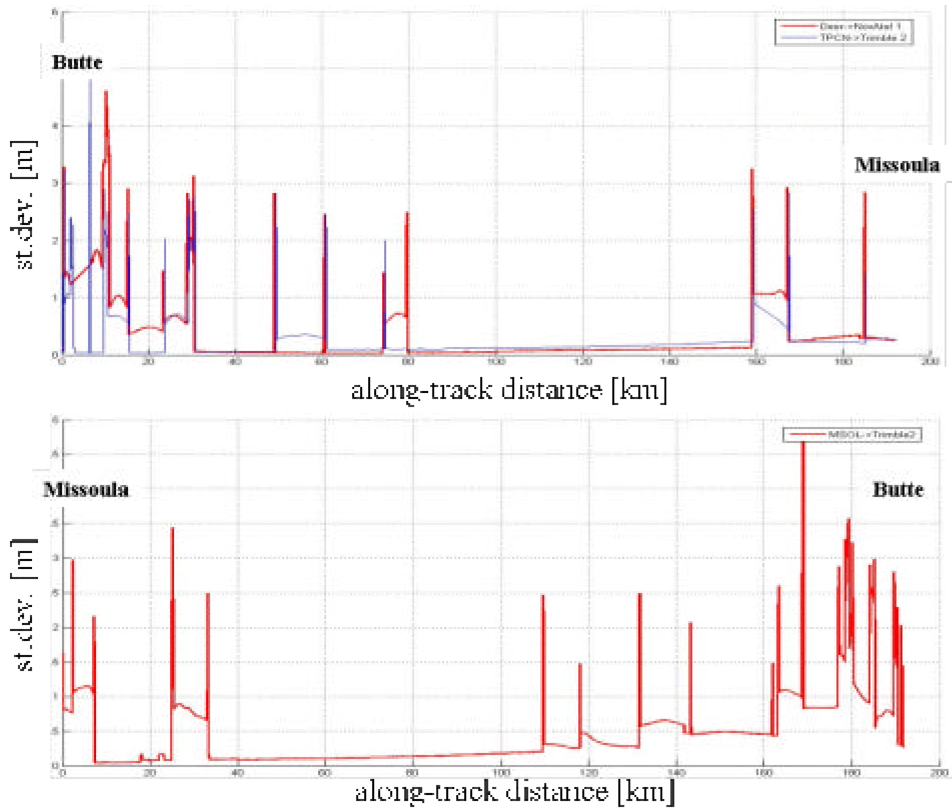
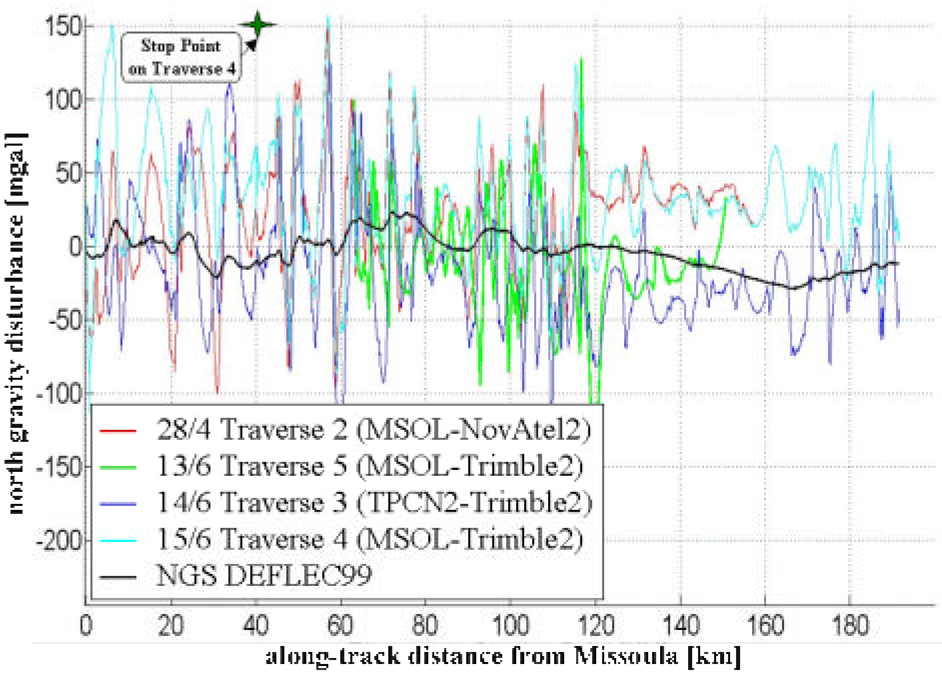
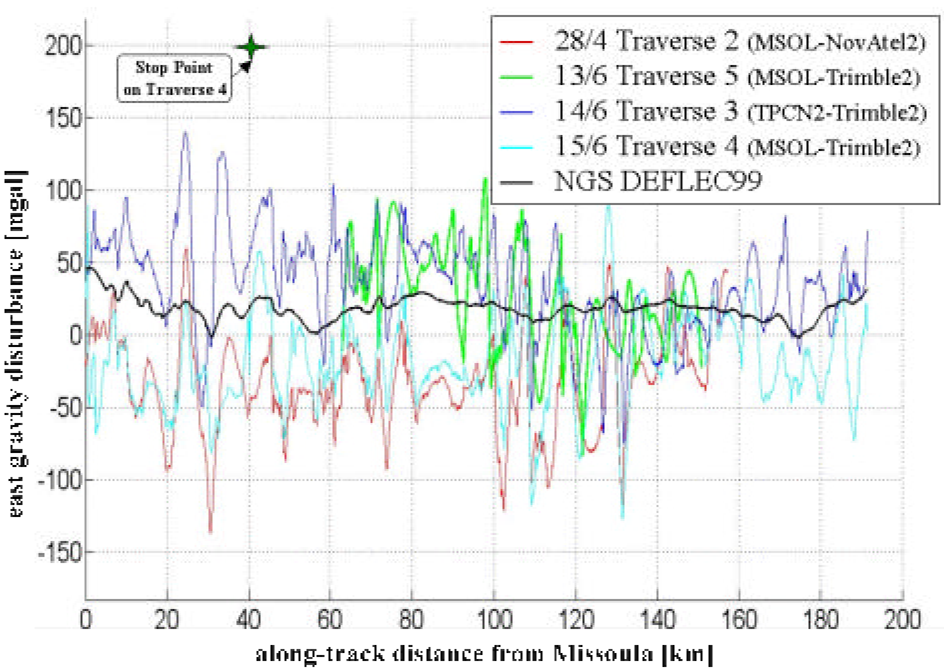



Figure 23: Predicted standard deviations of DGPS solutions for the GPSVan according to Applanix™ software. Top: Traverse 3 along Segment I90, 14 June 2005; bottom: Traverse 4 along Segment I90, 15 June 2005.

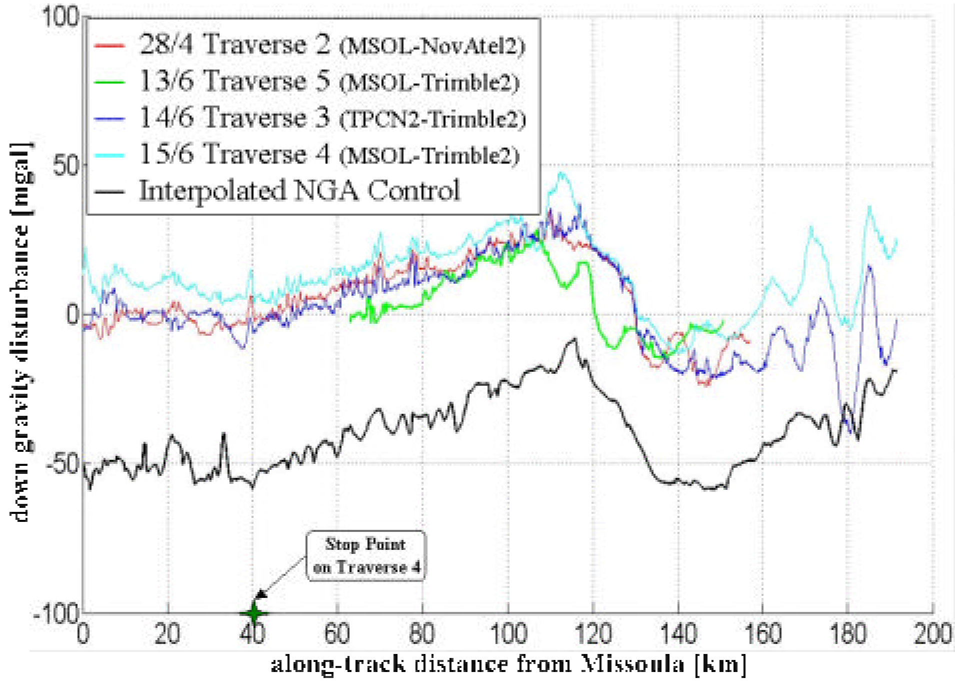
Estimations of the gravity disturbance vector are shown in Figures 24(abc) for the four traverses in comparison to control data. As before, we find clear (though not consistent) correlations in all three components among the traverses that are not evident with respect to the control profiles. However, the long-wavelength features in all three components agree well with the control. And, as before, we find significant errors (in terms of between-traverse comparisons) along the traverse near Butte, where the GPS standard deviations are greatest. In fact, the partial Traverse 5 on 13 June does not yield good results, particularly in the vertical component.


Stop Point
on Traverse 4
north gravity disturbance [mgal]
28/4 Traverse 2 (MSOL-NovAtel2)
13/6 Traverse 5 (MSOL-Trimble2)
14/6 Traverse 3 (TPCN2-Trimble2)
15/6 Traverse 4 (MSOL-Trimble2)
NGS DEFLEC99
along-track distance from Missoula [km]

a)


Stop Point
on Traverse 4
east gravity disturbance [mgal]
28/4 Traverse 2 (MSOL-NovAtel2)
13/6 Traverse 5 (MSOL-Trimble2)
14/6 Traverse 3 (TPCN2-Trimble2)
15/6 Traverse 4 (MSOL-Trimble2)
NGS DEFLEC99
along-track distance from Missoula [km]

b)




c)

Figure 24: Gravity vector estimates along Segment I90, from Missoula to Butte, using data collected on 28 April, 14-15 June, and 13 June 2005 (Traverses 2, 3, 4, and 5 respectively): a) north component, b) east component, c) down component. Values obtained from "Control" data and stop points are also indicated.

## 4. Summary, Conclusions, and Recommendations

The GPSVan gravimetric surveys in western Montana on 28 April and 13-15 June 2005 using INS and GPS can be characterized as a success. Over 700 km of roads were traversed, most road segments more than once. The repeated traverses essentially represent 4 segments along which we could analyze the quality of our gravity estimates using internal repeatability in addition to external control data. These are designated SR1 (along State Route 1), SR43 (along State Route 43), SR93 (along State Route 93), and I90 (along Interstate Route 90). On several sorties of the GPSVan, stops were included along the way at points where externally determined deflections of the vertical (DOV) and absolute gravity (or simply a GPS base station) were located. Otherwise, the traverses were run essentially at posted speeds (typically 80-100 km/hr) and the gravity estimations are based on an internal 180-second smoother, thus theoretically yielding a resolution (half-wavelength) of about 2-2.5 km. The suite of instruments consisted of (among other lower quality IMUs) three high-accuracy INS's (LN100, H764G1, and H764G2), as well as numerous GPS receivers. Unfortunately, only the H764G2 performed well enough to yield acceptable gravity estimates. Also, the Trimble2 (and NovAtel1 on 28 April) rover receivers performed consistently better than the others.

Of the 4 segments analyzed, SR1, SR43, and I90 yielded acceptable results, while SR93 can be classified as unacceptable. The determining factor (after the quality of the INS) is the quality of the GPS solution. If the INS is performing well (as was the case with the H764G2) and we assume that its quality is essentially constant along the traverse, then the accuracy in the GPS solution, as characterized by the predicted standard deviations in the GPS position solution, strongly correlates with the quality of the gravity estimates. This is the most important conclusion to be drawn from these tests and analyses. The inability to obtain acceptable gravity estimates along Segment SR93 is a direct consequence of the relatively poor quality of the GPS solution on this segment. It is noted that stop points have also caused degradation in the gravity estimates, although generally these points are not correlated with high GPS standard deviations. The problems in gravity estimation at these stop points may be due to the method of processing (continuously throughout the stationary period), and alternative methods that break the INS data stream into segments need to be explored.

Our estimates of the gravity disturbance vector are based on a well-tested Kalman filter algorithm (used successfully in airborne vector gravimetry), but does not include way-station control data. Thus the errors contain biases and trends that would need to be extracted for a final quality assessment. However, along uninterrupted sub-segments, where the GPS solution appears adequate, we may compute standard deviations of differences between estimates and control, or between estimates of different traverses. We compute these only for the down component since the errors appear to be generally of a random nature (the only case, strictly speaking, for which standard deviations make sense). The horizontal components, on the other hand, still contain systematic errors of more long-wavelength character, as well as possible scale errors (we expect horizontal components to have roughly the same magnitude as the vertical component; however the mountainous nature of western Montana may invalidate that general assumption). Tables 20 and 21 list the computed statistics for the indicated sub-segments.

Table 20: Statistics for the differences in the down component of the estimated gravity disturbance vector on SR1 and SR43. All values in units of mgal.

| Sub-Segment | Trav. 1 – Trav. 2 | | Trav. 1 – NGA control* | | Trav. 2 – NGA control* | |
|---|---|---|---|---|---|---|
| | mean | st. dev. | mean | st. dev. | mean | st.dev. |
| SR1, 0 – 40 km | -7.66 | **4.70** | 23.98 | **2.94** | 31.64 | **4.34** |
| SR43, 0 – 60 km | -3.57 | **2.07** | 32.95 | **2.23** | 36.51 | **2.70** |

* interpolated onto trajectory from point data

Table 21: Statistics for the differences in the down component of the estimated gravity disturbance vector on I90. All values in units of mgal.

| Difference, 45 – 95 km | mean | st. dev. |
|---|---|---|
| Trav. 2 – Trav. 3 | 3.17 | **2.40** |
| Trav. 2 – Trav. 4 | -5.28 | **0.72** |
| Trav. 3 – Trav. 4 | -8.45 | **2.74** |
| Trav. 2 – NGA Control* | 50.23 | **3.19** |
| Trav. 3 – NGA Control* | 47.06 | **2.95** |
| Trav. 4 – NGA Control* | 55.51 | **3.32** |

* interpolated onto trajectory from point data

The larger standard deviation between traverses for SR1 (Table 20) is due in part to the error in Traverse 2 caused by the stop point at the 19 km point; there is also an overall trend in their differences, which would be removable with minimal control data. For Segment I90 (Table 21), the between-traverse comparisons are generally better than the comparisons to the interpolated control data. In particular, Traverses 2 and 4, although run 1.5 months apart, from Missoula to Butte, refer to the same side of the divided highway, I-90, and agree to better than 1 mgal (st. dev.). Note that Traverse 3 from Butte to Missoula refers to the other side of I-90. Since these two sides are separated by up to a hundred meters, the more significant differences between oppositely run traverses shown in Table 21 should be expected. However, the differences with respect to the interpolated control show that the latter may not be adequate for a complete assessment of the quality of the estimates in this case. It is even more evident for the two (oppositely run) traverses along SR43, where the internal repeatability is much better than the agreement of either traverse with the interpolated control data (Table 20).

Certainly, the available control in the horizontal components seems almost completely deficient in light of the high correlation in estimates between repeated traverses. This supports the second important conclusion from these analyses. A proper assessment of the capability of this INS/GPS mobile gravimetry system requires dense control in all three components of the gravity disturbance vector along the actual road that was surveyed. A recommendation that arises from this conclusion is either to run the system along a road where such control already exists or to establish the requisite control along one of the segments surveyed, for example, either I90 or SR43. The latter could easily be done for the vertical component since these roads are by definition easily accessible and a night-time survey (to reduce the impact of traffic microseisms) could be readily accomplished. At the same time, one should consider the use of precise transportable astrolabe observations to obtain a true profile of the horizontal gravity disturbances, for example, using the equipment recently developed and proven by Hirt and Bürki (2002) and Hirt et al. (2004).

The success of the Montana survey resulted in part from the redundancy of the instrumentation. That is, had only one INS been used, we might have obtained very poor results indeed. On the other hand, from our initial experience with a test run in Ohio (along I-70 between Columbus and Dayton), we would have chosen the H764G2 as the most accurate candidate for further surveys. Nevertheless, this shows that redundancy (also in GPS receivers) is worthwhile when doing such extensive testing. In future tests one could use (as initially planned for his test) the redundancy in well performing instrumentation to cross-correlate two essentially independent solutions in order to filter out non-gravitational components (presumably due to systematic errors) and determine a final solution as if two traverses were run.

We found no particular advantage in the redundancy of GPS base stations. However, this bears further analysis once the GPS solutions are improved with better cycle ambiguity recovery. Also, utilization of the way point stops and incorporation of corresponding data to reduce the effect of accumulating systematic INS errors has not yet been investigated. This and the use of control data to remove biases and linear trends warrant further analysis and algorithm development. The potential scale error in the horizontal gravity disturbance estimates (which vary as much as 100-150 mgal over a distance of only 5 km) indicates that the states of the Kalman filter and their dynamics model may require some modification. However, this development would benefit greatly from a better assessment of the current estimation results using much improved control data. This would indicate more directly the range of adaptations that are required.

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## Appendix

Here we show a sample of the results for the H764G1 and LN100. Without exception, if the down component is not well estimated, the estimated horizontal components are worse. Therefore, results are shown only for the down component. Figure A-1 compares the estimates derived from the LN100 along the two I90 traverses of 28 April 2005. Clearly, in comparison to Figure 24c, these estimates show little internal repeatability and thus have huge errors. Errors in the horizontal components are of the order of 1000 mgal to 2000 mgal (not shown). The situation is slightly better, though not satisfactory, for H764G1 along Segment I90, as seen in Figure A-2. There is no repeatability between traverses and only overall agreement with the longer-wavelength control data. Again, the horizontal component estimates (not shown) are in error by hundreds of mgal. Figure A-3 compares the down component estimates from all three INS's along the entire trajectory from Butte to Missoula on 14 June 2005 (covering the individual Segments SR43 and SR93, as indicated). Again, the solution with LN100 fares poorly except at the long wavelengths, and the solution with H764G1 apparently contains larger high-frequency errors than the solution with H764G2. The standard deviation between estimates using H764G1 and H764G2 along the entire 295 km trajectory (Traverse 1) between Butte and Missoula is 8.2 mgal; with respect to the interpolated control it is 13.5 mgal (H764G1) and 15.3

mgal (H764G2). Corresponding standard deviations just for the 60 km part of Segment SR43 are 4.2 mgal, 4.0 mgal, and 2.2 mgal (see also Table 20).

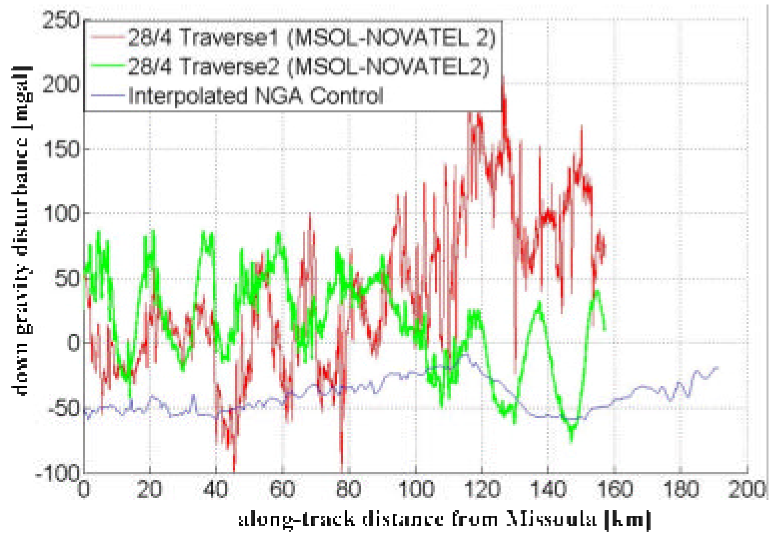


Figure A-1: Comparison of estimates of the down component of the gravity disturbance using the LN100 along I90 Traverses 1 and 2 of 28 April 2005.

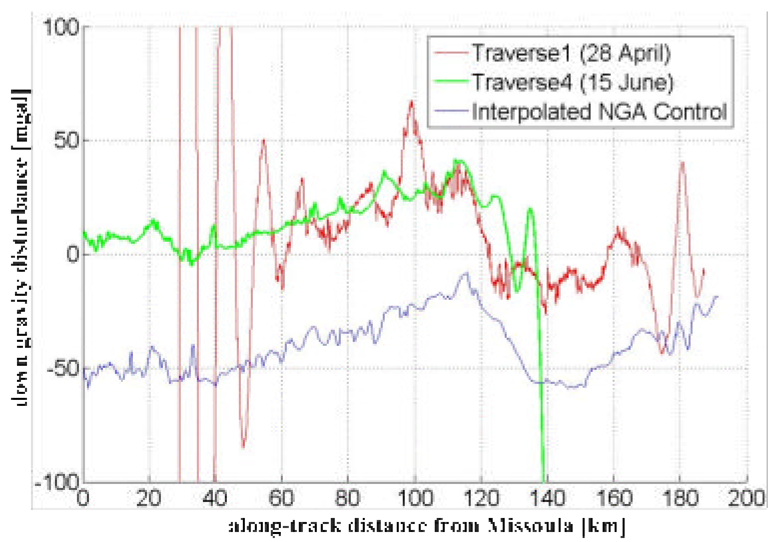


Figure A-2: Comparison of estimates of the down component of the gravity disturbance using the H764G1 along I90 Traverses 1 and 4 of 28 April and 15 June 2005, respectively.

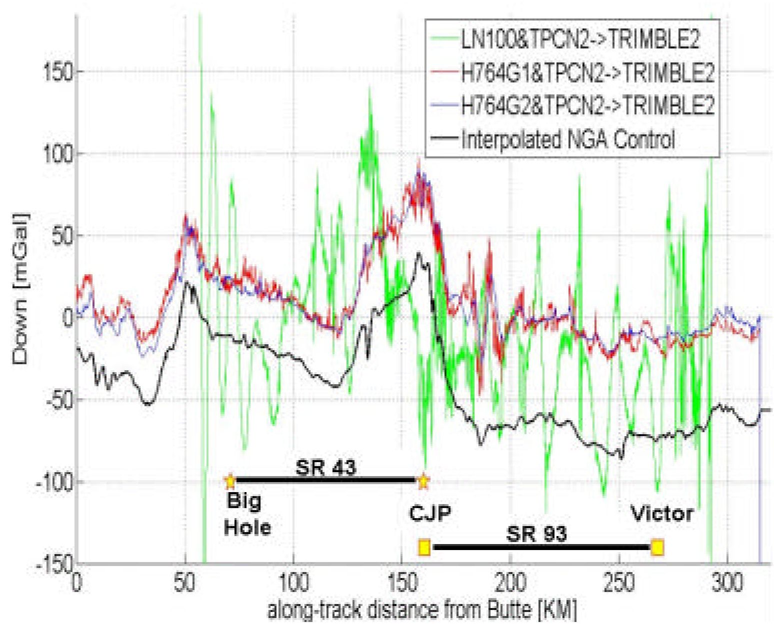


Figure A-3: Comparison of estimates of the down component of the gravity disturbance using the LN100, H764G1, and H764G2 along Routes 43 and 93 from Butte to Missoula (see Figure 2), on 14 June 2005.